

Interdisciplinary

**Statistics** 

## GENERALIZED LATENT VARIABLE MODELING

Multilevel, Longitudinal, and Structural Equation Models

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