

AN INTRODUCTION TO FLUID DYNAMICS

BY

G.K.BATCHELOR, F.R.S.

Professor of Applied Mathematics in the University of Cambridge

CAMBRIDGE
UNIVERSITY PRESS

CONTENTS

Preface *page* xiii

Conventions and Notation xviii

Chapter 1. The Physical Properties of Fluids

1.1	Solids, liquids and gases	i
1.2	The continuum hypothesis	4
1.3	Volume forces and surface forces acting on a fluid	7
	Representation of surface forces by the stress tensor, 9	
	The stress tensor in a fluid at rest, 12	
1.4	Mechanical equilibrium of a fluid	14
	A body 'floating' in fluid at rest, 16	
	Fluid at rest under gravity, 18	
1.5	Classical thermodynamics	20
1.6	Transport phenomena	28
	The linear relation between flux and the gradient of a scalar intensity, 30	
	The equations for diffusion and heat conduction in isotropic media at rest, 32	
	Molecular transport of momentum in a fluid, 36	
1.7	The distinctive properties of gases	37
	A perfect gas in equilibrium, 38	
	Departures from the perfect-gas laws, 45	
	Transport coefficients in a perfect gas, 47	
	Other manifestations of departure from equilibrium of a perfect gas, 50	
1.8	The distinctive properties of liquids	53
	Equilibrium properties, 55	
	Transport coefficients, 57	
1.9.	Conditions at a boundary between two media	60
	Surface tension, 60	
	Equilibrium shape of a boundary between two stationary fluids, 63	
	Transition relations at a material boundary, 68	

Chapter 2. Kinematics of the Flow Field

2.1	Specification of the flow field	71
	Differentiation following the motion of the fluid, 72	
2.2	Conservation of mass	73
	Use of a stream function to satisfy the mass-conservation equation, 75	
2.3	Analysis of the relative motion near a point	79
	Simple shearing motion, 83	

2.4	Expression for the velocity distribution with specified rate of expansion and vorticity	page 84
2.5	Singularities in the rate of expansion. Sources and sinks	88
2.6	The vorticity distribution	92
	Line vortices, 93	
	Sheet vortices, 96	
2.7	Velocity distributions with zero rate of expansion and zero vorticity	99
	Conditions for V_0 to be determined uniquely, 102	
	Irrotational solenoidal flow near a stagnation point, 105	
	The complex potential for irrotational solenoidal flow in two dimensions, 106	
2.8	Irrotational solenoidal flow in doubly-connected regions of space	108
	Conditions for V_0 to be determined uniquely, 112	
2.9	Three-dimensional flow fields extending to infinity	114
	Asymptotic expressions for ω , and u , 114	
	The behaviour of $\langle j \rangle$ at large distances, 117	
	Conditions for $V(j)$ to be determined uniquely, 119	
	The expression of $\langle j \rangle$ as a power series, 120	
	Irrotational solenoidal flow due to a rigid body in translational motion, 122	
2.10	Two-dimensional flow fields extending to infinity	124
	Irrotational solenoidal flow due to a rigid body in translational motion, 128	

Chapter 3. Equations Governing the Motion of a Fluid

3.1	Material integrals in a moving fluid	131
	Rates of change of material integrals, 133	
	Conservation laws for a fluid in motion, 135	
3.2	The equation of motion	137
	Use of the momentum equation in integral form, 138	
	Equation of motion relative to moving axes, 139	
3.3	The expression for the stress tensor	141
	Mechanical definition of pressure in a moving fluid, 141	
	The relation between deviatoric stress and rate-of-strain for a Newtonian fluid, 142	
	The Navier-Stokes equation, 147	
	Conditions on the velocity and stress at a material boundary, 148	
3.4	Changes in the internal energy of a fluid in motion	151
3.5	Bernoulli's theorem for steady flow of a frictionless non-conducting fluid	156
	Special forms of Bernoulli's theorem, 161	
	Constancy of H across a transition region in one-dimensional steady flow, 163	
3.6	The complete set of equations governing fluid flow	164
	Isentropic-flow, 165	
	Conditions for the velocity distribution to be approximately solenoidal, 167	
3.7	Concluding remarks to chapters i, 2 and 3	171

Chapter 4. Flow of a Uniform Incompressible Viscous Fluid

4.1	Introduction	page 174
	Modification of the pressure to allow for the effect of the body force,	176
4.2	Steady unidirectional flow	179
	Poiseuille flow,	180
	Tubes of non-circular cross-section,	182
	Two-dimensional flow,	182
	A model of a paint-brush,	183
	A remark on stability,	185
4.3	Unsteady unidirectional flow	186
	The smoothing-out of a discontinuity in velocity at a plane,	187
	Plane boundary moved suddenly in a fluid at rest,	189
	One rigid boundary moved suddenly and one held stationary,	190
	Flow due to an oscillating plane boundary,	191
	Starting flow in a pipe,	193
4.4	The Ekman layer at a boundary in a rotating fluid	195
	The layer at a free surface,	197
	The layer at a rigid plane boundary,	199
4.5	Flow with circular streamlines	201
4.6	The steady jet from a point source of momentum	205
4.7	Dynamical similarity and the Reynolds number	211
	Other dimensionless parameters having dynamical significance,	215
4.8	Flow fields in which inertia forces are negligible	216
	Flow in slowly-varying channels,	217
	Lubrication theory,	219
	The Hele-Shaw cell,	222
	Percolation through porous media,	223
	Two-dimensional flow in a corner,	224
	Uniqueness and minimum dissipation theorems,	227
4.9	Flow due to a moving body at small Reynolds number	229
	A rigid sphere,	230
	A spherical drop of a different fluid,	235
	A body of arbitrary shape,	238
4.10	Oseen's improvement of the equation for flow due to moving bodies at small Reynolds number	240
	A rigid sphere,	241
	A rigid circular cylinder,	244
4.11	The viscosity of a dilute suspension of small particles	246
	The flow due to a sphere embedded in a pure straining motion,	248
	The increased rate of dissipation in an incompressible suspension,	250
	The effective expansion viscosity of a liquid containing gas bubbles,	253
4.12	Changes in the flow due to moving bodies as R increases from 1. to about 100	255

Chapter 5. Flow at Large Reynolds Number: Effects of Viscosity

5.1	Introduction	page 264
5.2	Vorticity dynamics	266
	The intensification of vorticity by extension of vortex-lines, 270	
5.3	Kelvin's circulation theorem and vorticity laws for an inviscid fluid	273
	The persistence of irrotationality, 276	
5.4	The source of vorticity in motions generated from rest	277
5.5	Steady flows in which vorticity generated at a solid surface is prevented by convection from diffusing far away from it	282
	(a) Flow along plane and circular walls with suction through the wall, 282	
	(b) Flow toward a 'stagnation point' at a rigid boundary, 285	
	(c) Centrifugal flow due to a rotating disk, 290	
5.6	Steady two-dimensional flow in a converging or diverging channel	294
	Purely convergent flow, 297	
	Purely divergent flow, 298	
	Solutions showing both outflow and inflow, 301	
5.7	Boundary layers	302
5.8	The boundary layer on a flat plate	308
5.9	The effects of acceleration and deceleration of the external stream	314
	The similarity solution for an external stream velocity proportional to x^m, 316	
	Calculation of the steady boundary layer on a body moving through fluid, 318	
	Growth of the boundary layer in initially irrotational flow, 321	
5.10	Separation of the boundary layer	325
5.11	The flow due to bodies moving steadily through fluid	331
	Flow without separation, 332	
	Flow, with separation, 337	
5.12	Jets, free shear layers and wakes	343
	Narrow jets, 343	
	Free shear layers, 346	
	Wakes, 348	
5.13	Oscillatory boundary layers	353
	The damping force on an oscillating body, 355	
	Steady streaming due to an oscillatory boundary layer, 358	
	Applications of the theory of steady streaming, 361	

5.14 Flow systems with a free surface , page 364
 The boundary layer at a free surface, 364
 The drag on a spherical gas bubble rising steadily through liquid, 367
 The attenuation of gravity waves, 370

5.15 Examples of use of the momentum theorem , 372
 The force on a regular array of bodies in a stream, 372
 The effect of a sudden enlargement of a pipe, 373

Chapter 6. Irrotational Flow Theory and its Applications

6.1 The role of the theory of flow of an inviscid fluid 378

6.2 General properties of irrotational flow 380
 Integration of the equation of motion, 382
 Expressions for the kinetic energy in terms of surface integrals, 383
 Kelvin's minimum energy theorem, 384
 Positions of a maximum of q and a minimum of p , 384
 Local variation of the velocity magnitude, 386

6.3 Steady flow: some applications of Bernoulli's theorem and the momentum theorem 386
 Efflux from a circular orifice in an open vessel, 387
 Flow over a weir, 391
 Jet of liquid impinging on a plane wall, 392
 Irrotational flow which may be made steady by choice of rotating axes, 396

6.4 General features of irrotational flow due to a moving rigid body 398
 The velocity at large distances from the body, 399
 The kinetic energy of the-fluid, 402
 The force on a body in translational motion, 404
 The acceleration reaction, 407
 The force on a body in accelerating fluid, 409

6.5 Use of the complex potential for irrotational flow in two dimensions 409
 Flow fields obtained by special choice of the function $ui(z)$, 410
 Conformal transformation of the plane of flow, 413
 Transformation of a boundary into an infinite straight line, 418
 Transformation of a closed boundary into a circle, 420
 The circle theorem, 422

6.6 Two-dimensional irrotational flow due to a moving cylinder with circulation 423
 A circular cylinder, 424
 An elliptic cylinder in translational motion, 427
 The force and moment on a cylinder in steady translational motion, 433

6.7 Two-dimensional aerofoils 435
 The practical requirements of aerofoils, 435
 The generation of circulation round an aerofoil and the basis for Joukowski's hypothesis, 438
 Aerofoils obtained by transformation of a circle, 441
 Joukowski aerofoils, 444

6.8	Axisymmetric irrotational flow due to moving bodies	page 449
	Generalities, 449	
	A moving sphere, 452	
	Ellipsoids of revolution, 455	
	Body shapes obtained from source singularities on the axis of symmetry, 458	
	Semi-infinite bodies, 460	
6.9	Approximate results for slender bodies	463
	Slender bodies of revolution, 463	
	Slender bodies in two dimensions, 466	
	Thin aerofoils in two dimensions, 467	
6.10	Impulsive motion of a fluid	471
	Impact of a body on a free surface of liquid, 473	
6.11	Large gas bubbles in liquid	474
	A spherical-cap bubble rising through liquid under gravity, 475	
	A bubble rising in a vertical tube, 477	
	A spherical expanding bubble, 479	
6-12	Cavitation in a liquid	481
	Examples of cavity formation in steady flow, 482	
	Examples of cavity formation in unsteady flow, 485	
	Collapse of a transient cavity, 486	
	Steady-state cavities, 491	
6.13	Free-streamline theory, and steady jets and cavities	493
	Jet emerging from an orifice in two dimensions, 495 ^s	
	Two-dimensional flow past a flat plate with a cavity at ambient pressure, 497	
	Steady-state cavities attached to bodies held in a stream of liquid, 502	
Chapter 7. Flow of Effectively Inviscid Fluid with Vorticity		
7.1	Introduction	507
	The self-induced movement of a line vortex, 509	
	The instability of a sheet vortex, 511	
7.2	Flow in unbounded fluid at rest at infinity	517
	The resultant force impulse required to generate the motion, 518	
	The total kinetic energy of the fluid, 520	
	Flow with circular vortex-lines, 521	
	Vortex rings, 522	
7.3	Two-dimensional flow in unbounded fluid at rest at infinity	527
	Integral invariants of the vorticity: distribution, 528	
	Motion of a group of point vortices, 530	
	Steady motions, 532	
7.4	Steady two-dimensional flow with vorticity throughout the fluid	536
	Uniform vorticity in a region bounded externally, 538	
	Fluid in rigid rotation at infinity, 539	
	Fluid in simple shearing motion at infinity, 541	

7.5	Steady axisymmetric flow with swirl	page 543
	The effect of a change of cross-section of a tube on a stream of rotating fluid, 546	
	The effect of a change of external velocity on an isolated vortex, 550	
7.6	Flow systems rotating as a whole	555
	The restoring effect of Coriolis forces, 555	
	Steady flow at small Rossby number, 557	
	Propagation of waves in a rotating fluid, 559	[^] <i>i</i>
	Flow due to a body moving along the axis of rotation, 564	
7.7	Motion in a thin layer on a rotating sphere	567
	Geostrophic flow, 571	
	Flow over uneven ground, 573	
	Planetary waves, 577	
7.8	The vortex system of a wing	580
	General features of the flow past lifting bodies in three dimensions, 580	
	Wings of large aspect ratio, and 'lifting-line' theory, 583	
	The trailing vortex system far downstream, 589	
	Highly swept wings, 591	

Appendices

Measured values of some physical properties of common fluids	594
(a) Dry air at a pressure of one atmosphere, 594	
(b) The Standard Atmosphere, 595	
(c) Pure water, 595	
(d) Diffusivities for momentum and heat at 15 °C and 1 atm, 597	
(e) Surface tension between two fluids, 597	
Expressions for some common vector differential quantities in orthogonal curvilinear co-ordinate systems	598
<i>Publications referred to in the text</i>	604
<i>Subject Index</i>	609

Plates 1 to 24 are between pages 364 and 365