

AEROTHERMODYNAMICS OF TURBOMACHINERY

ANALYSIS AND DESIGN

Naixing Chen

Institute of Engineering Thermophysics, Chinese Academy of Sciences, China



John Wiley & Sons (Asia) Pte Ltd

Contents

Foreword		xv
Preface	*	xyii
Acknowledgments		xix
Nomenclature		xxi
1 Introduction		
1.1 Introduction to the Study of the Aerothermodynamics of Turbomachinery		
1.2 Brief Description of the Development of the Numerical Study of the Aerothermodynamics of Turbomachinery		
1.3 Summary		
Further Reading		
Governing Equations Expressed in Non-Orthogonal Curvilinear Coordinates to Calculate 3D Viscous Fluid Flow in Turbomachinery		9
2.1 Introduction		9
2.2 Aerothermodynamics Governing Equations (Navier-Stokes Equations) of Turbomachinery		10/
2.3 Viscous and Heat Transfer Terms of Equations		11
2.3.1 Viscous Stress Tensor		,12
2.3.2 Strain Tensor		
2.3.3 Viscous Force		14
2.3.4 Rates of Work Done by the Viscous Stresses and Dissipation Function		14
2.3.5 Heat Transfer Term	x	15
2.4 Examples of Simplification of Viscous and Heat Transfer Terms		15
2.4.1 Three-Dimensional Flow in Turbomachinery Expressed by Using Arbitrary Non-Orthogonal Coordinates		15
2.4.2 SI Stream-Surface Flow		17
2.4.3 S2 Stream-Surface Flow		17
2.4.4 Annulus Wall Boundary Layer		17
2.4.5 Three-Dimensional Boundary Layer on Rotating Blade Surface		19
2.5 Tensor Form of Governing Equations		20
2.5.1 Continuity Equation		20
2.5.2 Momentum Equation		20

2.5.3	Energy Equation	21
2.5.4	Entropy Equation	21
2.6	Integral Form of Governing Equations	21
2.6.1	Continuity Equation	21
2.6.2	Momentum Equation	21
2.6.3	Energy Equation	21
2.7	A Collection of the Basic Relationships for Non-Orthogonal Coordinates	22
2.8	Summary	2^
Introduction to Boundary Layer Theory		25
3.1	Introduction	25
3.2	General Concepts of the Boundary Layer	25
3.2.1	Nature of Boundary Layer Flow	25
3.2.2	Boundary Layer Thicknesses	27
3.2.3	Transition of the Boundary Layer Regime	29
3.2.4	Boundary Layer Separation	30
3.2.5	Thermal Boundary Layer	32
3.3	Summary	35
Numerical Solutions of Boundary Layer Differential Equations		37
4.1	Introduction	37
4.2	Boundary Layer Equations Expressed in Partial Differential Form	37
4.2.1	Two-Dimensional Laminar Boundary Layer Equations	37
4.2.2	Laminar Boundary Layer Equations of Axisymmetrical Flow	38
4.2.3	Turbulent Boundary Layer Equations	39
4.2.4	Boundary Conditions of Solution	40
4.3	Numerical Solution of the Boundary Layer Differential Equations for a Cascade on the Stream Surface of Revolution	41
4.3.1	Boundary Layer Equations of SI Stream Surface Flow of Revolution and Their Solution	41
4.3.2	Turbulence Modeling	44
4.4	Calculation Results and Validations	45
4.4.1	Laminar Boundary Layer Calculation Example	45
4.4.2	Turbulent Boundary Layer with Favorable Pressure Gradient	45
4.4.3	Turbulent Boundary Layer with Adverse Pressure Gradient (Ludweig and Tillmann)	46
4.4.4	Turbulent Boundary Layer with Favorable Pressure Gradient (Bell)	47
4.4.5	Turbulent Boundary Layer with Adverse Pressure Gradient (Schubauer and Spangenberg)	48
4.5	Application to Analysis of the Performance of Turbomachinery Blade Cascades	49
4.5.1	Boundary Layer Momentum Thickness (Bammert's Experiment)	49
4.5.2	Laminar Boundary Layer Prediction (Turbine and Compressor Blade Profiles)	49
4.5.3	Laminar-Turbulent Boundary Layer Prediction	51
4.5.4	Turbulent Viscosity Prediction	52

4.5.5	Stagger Angle Effect (C4)	53.
4.5.6	Effect of Incidence Angle on Blade Loss Coefficient (C4)	55
4.5.7	Effect of Reynolds Number on the Loss Coefficient of Compressor Blade Cascades.(C4)	55
4.5.8	Effect of Stream Sheet Thickness on Boundary Layer Momentum Thickness (Turbine Blade)	55
4.6.	Summary	57
Approximate Calculations Using Integral Boundary Layer Equations		59
5.1	Introduction	59
5.2	Integral Boundary Layer Equations	59
5.2.1	Boundary Layer Momentum Integral Equation of the Flow on the Stream Surface of Revolution	59
5.2.2	Momentum and Energy Integral Equations of the Boundary Layer for Different Flow Cases	62
5.3	Generalized Method for Approximate Calculation of the Boundary Layer Momentum Thickness	64
5.4	Laminar Boundary Layer Momentum Integral Equation	66
5.5	Transitional Boundary Layer Momentum Integral Equation	68
5.5.1	Velocity Distribution in the Boundary Layer Region	68
5.5.2	Wall Shear Stress Prediction in the Transitional Region	68
5.5.3	An Approximate Momentum Integral Equation for the Transitional Region	70
5.6	Turbulent Boundary Layer Momentum Integral Equation	70
5.6.1	The Law of Velocity Distribution	71
5.6.2	Shape Parameters, H and H_{20}	72
5.6.3	Wall Shear Stress Coefficient	72
5.6.4	Boundary Layer Momentum Thickness Prediction	75
5.6.5	An Approximate Formula for Prediction of the Shape Parameter H of the Turbulent Boundary Layer	78/
5.6.6	Empirical Constants for the Generalized Method for Approximate Calculation of Turbulent Boundary Layer Momentum Thickness Proposed by Different Authors	80
5.7	Calculation of a Compressible Boundary Layer	81
5.7.1	Compressibility Transformation of the Integral Equation of the Boundary Layer	81
5.7.2	Calculation Method for a Compressible Boundary Layer Without Heat Transfer	83
5.7.3	Boundary Layer Calculation Method for a Blade Cascade on the Stream Surface of Revolution	84
5.8	Summary	84
Application of Boundary Layer Techniques to Turbomachinery		87
6.1	Introduction	87
6.2	Flow Rate Coefficient and Loss Coefficient of Two-Dimensional Blade Cascades	87

6.2.1	Flow Rate Coefficient of a Blade Cascade	88
6.2.2	Loss Coefficient of a Blade Cascade	89
6.3	Studies on the Velocity Distributions Along Blade Surfaces and Correlation Analysis of the Aerodynamic Characteristics of Plane Blade Cascades	j
6.3.1	Influence of Blade Surface Velocity Distribution on Boundary Layer Momentum Loss Thickness	j
6.3.2	The Loss Coefficient of a Theoretical Optimum Plane Turbine Profile Cascade	j
6.3.3	Correlations of the Loss Coefficient of a Plane Turbine Profile Cascade (Using the Geometrical Convergence Gradient of Blade Passage, G)	\
6.3.4	Correlations of the Loss Coefficient of a Plane Turbine Profile Cascade (Using the Convergence Gradient of Blade Passage G^{**} Expressed by Flow Angles)	j
6.3.5	Correlations of the Loss Coefficient of a Plane Compressor Blade Cascade (Using Diffusion Factor D)	j
6.4	Summary	99
7	Stream Function Methods for Two- and Three-Dimensional Flow Computations in Turbomachinery	101
7.1	Introduction	103
7.2	Three-Dimensional Flow Solution Methods with Two Kinds of Stream Surfaces	\
7.2.1	Three-Dimensional Solution	104
7.2.2	Quasi-Three-Dimensional Solution	106
7.3	Two-Stream Function Method for Three-Dimensional Flow Solution	106
7.3.1	Coordinate System and Metrical Tensors	106
7.3.2	Three-Dimensional Governing Equations of Steady Inviscid Fluid Flow	109.
7.3.3	Definition of Stream Functions and Coordinate-Transformation	i
7.3.4	Boundary Conditions and Calculation Examples	110
7.4	Stream Function Methods for Two-Dimensional Viscous Fluid Flow Computations	j
7.4.1	Navier-Stokes Equation Solution for Rotating Blade Cascade Flow on an SI Stream Surface of Revolution	M^
7.4.2	Boundary Conditions	
7.4.3	Solution Procedure	x
7.4.4	Calculation Examples	119
7.5	Stream Function Method for Numerical Solution of Transonic Blade Cascade Flow on the Stream Surface of Revolution	122
7.5.1	Stream Function Equation and Artificial Compressibility	123
7.5.2	Stone's Strongly Implicit Procedure (SIP) and its Improvement	128
7.5.3	Numerical Solution Procedure	129
7.5.4	Calculation Examples	130
7.6	Finite Analytic Numerical Solution Method (FASM) for Solving the Stream Function Equation of Blade Cascade Flow	131

7.6.1	Governing Equation and its Solution	132
7.6.2	Linearization of Equation Solution for a Rectangular Region	133
7.6.3	Non-Orthogonal Coordinate System and Discretized Difference Equation	134
7.6.4	Adaptability of the Coefficients to Compressibility	136
7.6.5	Numerical Solution Procedure	137
7.6.6	Calculation Examples	137
7.7	Summary	140
Appendix 7.A	Formulas for Estimating the Coefficients of the Differential Equations of the 3D Two-Stream Function Coordinate Method	141
8	Pressure Correction Method for Two-Dimensional and Three-Dimensional Flow Computations in Turbomachinery	145
8.1	Introduction	145
8.2	Governing Equations of Three-Dimensional Turbulent Flow and the Pressure Correction Solution Method	I 146
8.2.1	Governing Equations	146
8.2.2	Two-Equation ($k - \epsilon$) Turbulence Model	148
8.2.3	Coordinate Transformation and Generalized Form of Governing Equations with Body-Fitted Coordinates for Calculating Orthogonal Coordinate Components of the Velocity Vector	150
8.2.4	Discretized Algebraic Equations	151
8.2.5	Boundary Conditions and Wall-Function Treatment	156
8.3	Two-Dimensional Turbulent Flow Calculation Examples	157
8.3.1	A Symmetric Airfoil	157
8.3.2	Low Speed Subsonic Turbine Blade Cascade (NACA TN-3802)	159
8.3.3	Turbine Blade Cascade (VKI-LS59)	162
8.3.4	Transonic Turbine Blade Cascade with Large Round Leading Edges (T12)	165
8.3.5	Supersonic Turbine Blade Cascade	166
8.3.6	Compressor Blade Cascade (T1)	166
8.4	Three-Dimensional Turbulent Flow Calculation Examples	169
8.4.1	Linear Turbine Blade Cascade	173
8.4.2	Annular Turbine Blade Cascade	175
8.4.3	High Turning Turbine Blade Cascade for an Annular ^x Blade Cascade Wind Tunnel	181
8.4.4	Linear Compressor Cascade	183
8.4.5	BUAA Single Rotor Test Compressor	185
8.4.6	Centrifugal Impeller	192
8.5	Summary	198
	Time-Marching Method for Two-Dimensional and Three-Dimensional Flow Computations in Turbomachinery	199
9.1	Introduction	199
9.2	Governing Equations of Three-Dimensional Viscous Flow in Turbomachinery	201

9.2.1	Relative Motion in Turbomachinery	201
9.2.2	Governing Equations Written in Differential Equation Formulation	201
9.2.3	Governing Equations Written in Integral Form	204
9.3	Solution Method Based on Multi-Stage Runge-Kutta Time-Marching Scheme	j 205
9.3.1	Discretization of Governing Equations	205
9.3.2	Method for Prediction of Parameters on Boundary Surfaces and Fluxes	: 205
9.3.3	Adaptive Dissipation Term	206
9.3.4	Modified Multi-Stage Runge-Kutta Time-Marching Scheme	208
9.3.5	Turbulence Modeling and Wall Function	213
9.3.6	Multi-Grid Scheme	215
9.4	Two-Dimensional Turbulent Flow Examples Calculated by the Multi-Stage Runge-Kutta Time-Marching Method	j 216
9.4.1	A Grid Generation Method Based on Analogy with the Staff-Spring System	j 216
9.4.2	Turbine Blade Cascade (VKI-LS59)	220
9.4.3	Transonic Steam Turbine Blade Cascade (VKI-LS59 ST)	221
9.4.4	Supersonic Inlet Flow Compressor Blade Cascade	225
9.5	Three-Dimensional Flow Examples Calculated by the Multi-Stage Runge-Kutta Time-Marching Method	226
9.5.1	Numerical Solution for Three-Dimensional Inviscid Flow in a Transonic Single Rotor Compressor	, I 227
9.5.2	Numerical Solution for Three-Dimensional Turbulent Flow in a Single Rotor Compressor	i 232
9.5.3	Numerical Solution for Three-Dimensional Turbulent Flow in a Turbine Stage	233
9.5.4	Three-Dimensional Turbulent Flow in a Centrifugal Impeller by the Modified Multi-Stage Runge-Kutta Time-Marching Method	. 238
9.6	Summary	249
10	Numerical Study on the Aerodynamic Design of Circumferential- and Axial-Leaned and Bowed Turbine Blades	' 251
10.1	Introduction	. 2 5 1
10.2	Circumferential Blade-Bowing Study	• ' -x 252
10.2.1	Circumferential Blade-Bowing Procedure	252
10.2.2	Effect on the Pressure Distributions of the Surfaces of Revolution at Different Span Heights	I , 255
10.2.3	Effect on Parameter Contours of the Meridian Surfaces ($x^2 = \text{const}$)	. - 25{7
10.2.4	Effect on Pressure Contours of the Coordinate Surfaces ($x^1 = \text{const}$)	258
10.2.5	The Bowing Effect for Restraining Boundary-Layer Separation from the End-Wall	! 260

10.2.6	Circumferential Bowing Effect on Pitch-Wise Mass-Averaged Parameters at Station 3	I i	261
10.2.7	Suggestion of Applying a New Circumferentially Bowed Blading	: i	266
10.3	Axial Blade-Bowing Study	! :	266
10.3.1	Axial Blade-Bowing Procedure	:	266
10.3.2	Effect on Static Pressure Contours of the Meridian Surfaces ($x^2 = \text{const}$)	i :	268
10.3.3	Effect on Pressure Distributions of the Surfaces of Revolution at Different Span Heights	; !	269
10.3.4	Effect on Static Pressure Contours of the Surfaces of $x^j = \text{const}$: :	271
10.3.5	Effect on Circumferentially Averaged Parameters at the Vertical Measuring Plane (Just at the Exit from the Blade Channel, that is, Station No. 3)	j j	271
10.3.6	Axial Bowing Effect on Secondary Flow		
10.3.7	Axial Bowing Effect on Global Adiabatic Efficiency and Flow Rate		
10.4	Circumferential Blade-Bowing Study of Turbine Nozzle Blade Row with Low Span-Diameter Ratio		
10.4.1	Leaning Effect on Adiabatic Efficiency and Exit Flow Angle		
10.4.2	Generation of a Radial Stacking Form Close to Optimal		
10.4.3	An Attempt at a Blade Modification		
10.5	Summary		

11 Numerical Study on Three-Dimensional Flow Aerodynamics and Secondary Vortex Motions in Turbomachinery

11.1	Introduction		
11.2	Post-Processing Algorithms		
11.2.1	Relative Velocity Vector Schemes, Surface Trace and Volume Trace		
11.2.2	Vortex Intensity		
11.2.3	Entropy Increment		
11.2.4	An Approximate Formula for Predicting the Secondary Flow Velocity Vector		
11.3	Axial Turbine Secondary Vortices		
11.3.1	Saddle Point and Horseshoe Vortex		
11.3.2	Bowing Effect on the Location of the Saddle Point		
11.3.3	Passage Vortex		
11.3.4	Bowing Effect on the Development of the Passage Vortex		
11.3.5	Bowing Effect on the Passage Vortex for Different Incidence Angles		
11.3.6	Corner Vortex in Straight and Saber-Shaped Blade Cascades		
11.3.7	Tip Clearance Vortex		
11.3.8	Blade Bowing Effect in Blades with Tip Clearance		
11.3.9	Mechanism of Loss Reduction by Bowed Blades		

11.4	Some Features of Straight-Leaned Blade Aerodynamics of a Turbine Nozzle with Low Span-Diameter Ratio	310
11.4.1	Leaning Effect on Static Pressure Contours on the Blade Surfaces and on the Exit Coordinate Plane	j 310
11.4.2	Leaning Effect on Limiting Streamlines on Blade Surfaces	311
11.4.3	Leaning Effect on Entropy Contours at the Exit Plane from the Blade Channel	i 311
11.5	Numerical Study on the Three-Dimensional Flow Pattern and Vortex Motions in a Centrifugal Compressor Impeller	j 317
11.5.1	Complexity of the Flow in an Impeller	317
11.5.2	Limiting Streamlines on the Pressure/Hub and Suction/Hub Surfaces	317
11.5.3	Secondary Vortices in the Centrifugal Impeller	319
11.5.4	Topology of the Passage Vortex in the Centrifugal Compressor Impeller	j 322
11.5.5	Separation Vortex in a Vaneless Diffuser	325
11.5.6	Vaneless Diffuser Design Improvement	326
11.6	Summary	326
12	Two-Dimensional Aerodynamic Inverse Problem Solution Study in Turbomachinery	i 329
12.1	Introduction	329
12.2	Stream Function Method	331
12.2.1	S2 Meridional Stream Surface Flow	332
12.2.2	SI Stream Surface Flow of Revolution	334
12.3	A Hybrid Problem Solution Method Using the Stream Function Equation with Prescribed Target Velocity for the Blade Cascades of Revolution	j 336
12.3.1	Circumferentially Geometric Proportional Curvilinear Coordinate System	337
12.3.2	Stream Function Equation and its Coefficients	338
12.3.3	Solution Procedure	339
12.3.4	Calculation Examples	340 \
12.4	Stream-Function-Coordinate Method (SFC) for the Blade Cascades on the Surface of Revolution	' 343
12.4.1	Stream-Function-Coordinate Equation	' 343
12.4.2	Artificial Compressibility Technique	: , 345
12.4.3	Boundary Conditions	*x 345
12.4.4	Numerical Examples	346
12.5	Stream-Function-Coordinate Method (SFC) with Target Circulation for the Blade Cascades on the Surface of Revolution	350
12.5.1	Blade Circulation and its Derivative	351
12.5.2	Blade Thickness Distribution	/ 352
12.6	Two-Dimensional Inverse Method Using a Direct Solver with Residual Correction Technique	j 353
12.6.1	Residual Correction Equation	354
12.6.2 \	A Calculation Example	357
12.7	Summary	359

13	Three-Dimensional Aerodynamic Inverse Problem Solution Study in Turbomachinery	361
13.1	Introduction	361
13.2	Two-Stream-Function-Coordinate-Equation Inverse Method	362
13.2.1	Two-Stream-Function-Coordinate Differential Equations	362
13.2.2	Inverse Problem Solution Procedure	363
13.2.3	A Calculation Example	363
13.3	Three-Dimensional Potential Function Hybrid Solution Method	364
13.3.1	Governing Equations	364
13.3.2	Potential Function Equation	366
13.3.3	Solution Procedure	366
13.3.4	Calculation Example	L370
13.4	Summary	372
14	Aerodynamic Design Optimization of Compressor and Turbine Blades	375
14.1	Introduction	375
14.2	Parameterization Method	b77
14.2.1	Parameterization of Blade Profile and Stacking Line	-1378
14.2.2	2D Blade Reconstruction (Rebuilding)	1381
14.2.3	Parameter Effects on the Geometry of a Blade Profile	384
14.3	Response Surface Method (RSM) for Blade Optimization	'387
14.3.1	Response Surface Creation	394
14.3.2	Principle Scheme of the Response Surface Method	395
14.4	A Study on the Effect of Maximum Camber Location for a Transonic Fan Rotor Blading by GPAM	395
14.4.1	Brief Description	396
14.4.2	Optimization Procedure	397 "
14.5	Optimization of a Low Aspect Ratio Turbine by GPAM and a Study of the Effects of Geometry on the Aerodynamics Performance	I
14.5.1	Geometry Effect on Blade Performance	kpi
14.5.2	Optimal Turbine Nozzle Blades	1403
14.6	Blade Parameterization and Aerodynamic Design Optimization for a 3D Transonic Compressor Rotor	412
14.6.1	Calculation Example	J413
14.6.2	Brief Description of Methodologies	J414
14.6.3	Optimization with Response Surface Method (RSM)	J417
14.6.4	Optimization by Gradient-Based Parameterization Method (GPAM)	I
14.6.5	Simple Gradient Method (SGM)	419
14.6.6	Final Results	422
14.7	Summary	423
		426
	References	429
	Index	441