

Real Analysis

AN INTRODUCTORY COURSE

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* Sections marked with an asterisk need not be studied at this point but can be postponed until a later point in the course.

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