

Electromagnetic Theory of Gratings

Edited by R. Petit

With Contributions by

**L. C. Botten M. Cadilhac G. H. Derrick
D. Maystre R. C. McPhedran M. Nevière
R. Petit P. Vincent**

With 182 Figures

Springer-Verlag Berlin Heidelberg New York 1980

Contents

1. A Tutorial Introduction. By R. Petit (With 13 Figures)	1
1.1 Preliminaries	1
1.1.1 General Notations	1
1.1.2 Time-Harmonic Maxwell Equations	2
1.1.3 Boundary Conditions	3
1.1.4 Electromagnetism and Distribution Theory	4
1.1.5 Notations Used in the Description of a Grating	5
1.2 The Perfectly Conducting Grating	6
1.2.1 Generalities	6
1.2.2 The Diffracted Field	7
1.2.3 The Rayleigh Expansion and the Grating Formula	9
1.2.4 An Important Lemma	11
1.2.5 The Reciprocity Theorem	12
1.2.6 The Conservation of Energy	13
1.2.7 The Littrow Mounting	14
1.2.8 The Determination of the Coefficients B_n by the Rayleigh Method	15
1.2.9 An Integral Expression of u^d in P Polarization	19
1.2.10 The Integral Method in P Polarization	24
1.2.11 The Integral Method in S Polarization	26
1.2.12 Modal Expansion Methods	26
1.2.13 Conical Diffraction	31
1.3 The Dielectric or Metallic Grating	33
1.3.1 Generalities	33
1.3.2 The Diffracted Field Outside the Groove Region	33
1.3.3 Maxwell Equations and Distributions	35
1.3.4 The Principle of the Differential Method (in P Polarization) ..	35
1.4 Miscellaneous	39
References	40
Appendix A: The Distributions or Generalized Functions	41
A.1 Preliminaries	42
A.2 The Function Space \mathcal{R}	43
A.3 The Space \mathcal{R}^1	43

A.3.1 Definitions	43
A.3.2 Examples of Distributions	43
A.4 Derivative of a Distribution	44
A.5 Expansion with Respect to the Basis $e_j(x) = \exp[i(nK+k \sin\theta)x] = \exp(i\alpha_n x)$	47
A.5.1 Theorem	47
A.5.2 Proof	48
A.5.3 Application to δ_R	49
A.6 Convolution	49
A.6.1 Memoranda on the Product of Convolution in D'_1	49
A.6.2 Convolution in R'	50
 2. Some Mathematical Aspects of the Grating Theory. By M. Cadilhac	53
2.1 Some Classical Properties of the Helmholtz Equation	53
2.2 The Radiation Condition for the Grating Problem	54
2.3 A Lemma	55
2.4 Uniqueness Theorems	56
2.4.1 Metallic Grating, with Infinite Conductivity	56
2.4.2 Dielectric Grating	57
2.5 Reciprocity Relations	58
2.6 Foundation of the Yasuura Improved Point-Matching Method	59
2.6.1 Definition of a Topological Basis	59
2.6.2 The System of Rayleigh Functions is a Topological Basis	60
2.6.3 The Convergence of the Rayleigh Series; A Counterexample	61
References	62
 3. Integral Methods. By D. Maystre (With 8 Figures)	63
3.1 Development of the Integral Method	63
3.2 Presentation of the Problem and Intuitive Description of an Integral Approach	65
3.2.1 Presentation of the Problem	65
3.2.2 Intuitive Description of an Integral Approach	66
3.3 Notations, Mathematical Problem and Fundamental Formulae	67
3.3.1 Notations and Mathematical Formulation	67
3.3.2 Basic Formulae of the Integral Approach	69
3.4 The Uncoated Perfectly Conducting Grating	71
3.4.1 The TE Case of Polarization	72
3.4.2 The TM Case of Polarization	74
3.5 The Uncoated Dielectric or Metallic Grating	76
3.5.1 The Mathematical Boundary Problem	76
3.5.2 Vital Importance of the Choice of a Well-Adapted Unknown Function	77

3.5.3 Mathematical Definition of the Unknown Function and Determination of the Field and Its Normal Derivative Above P	77
3.5.4 Expression of the Field in M_2 as a Function of ϕ	79
3.5.5 Integral Equation	79
3.5.6 Limit of the Equation when the Metal Becomes Perfectly Conducting	80
3.6 The Multiprofile Grating	81
3.7 The Grating in Conical Diffraction Mounting	85
3.8 Numerical Application	89
3.8.1 A Fundamental Preliminary Choice	89
3.8.2 Study of the Kernels	90
3.8.3 Integration of the Kernels	93
3.8.4 Particular Difficulty Encountered with Materials of High Conductivity	96
3.8.5 The Problem of Edges	98
3.8.6 Precision on the Numerical Results	98
References	100
4. <i>Differential Methods.</i> By P. Vincent (With 11 Figures)	101
4.1 Introductory Remarks	102
4.1.1 Historical Survey	102
4.1.2 Definition of Problem	102
4.2 The E_{\parallel} Case	103
4.2.1 The Reflection and Transmission Matrices	104
4.2.2 The Computation of Transmission and Reflection Matrices	105
4.2.3 Numerical Algorithms	106
4.2.4 Alternative Matching Procedures for Some Grating Profiles	108
4.2.5 Field of Application	108
4.3 The H_{\parallel} Case	109
4.3.1 The Propagation Equation	109
4.3.2 Numerical Treatment	110
4.3.3 Field of Application	111
4.4 The General Case (Conical Diffraction Case)	111
4.4.1 The Reflection and Transmission Matrices	112
4.4.2 The Differential System	112
4.4.3 Matching with Rayleigh Expansions	114
4.4.4 Field of Application	114
4.5 Stratified Media	115
4.5.1 Stack of Gratings	115
4.5.2 Plane Interfaces Between Homogeneous Media	116

4.6	Infinitely Conducting Gratings: the Conformal Mapping Method	117
4.6.1	Method	117
4.6.2	Determination of the Conformal Mapping	119
4.6.3	Field of Application	121
	References	121
5.	<i>The Homogeneous Problem.</i> By M. Nevière (With 25 Figures)	123
5.1	Historical Summary	124
5.2	Plasmon Anomalies of a Metallic Grating	126
5.2.1	Reflection of a Plane Wave on a Plane Interface	126
5.2.2	Reflection of a Plane Wave on a Grating	130
5.3	Anomalies of Dielectric Coated Reflection Gratings Used in TE Polarization	136
5.3.1	Determination of the Leaky Modes of a Dielectric Slab Bounded by Metal on One of Its Sides	137
5.3.2	Reflection of a Plane Wave on a Dielectric Coated Reflection Grating Used in TE Polarization	140
5.4	Extension of the Theory	143
5.4.1	Anomalies of a Dielectric Coated Grating Used in TM Polarization	143
5.4.2	Plasmon Anomalies of a Bare Grating Supporting Several Spectral Orders	145
5.4.3	General Considerations on Anomalies of a Grating Supporting Several Spectral Orders	148
5.5	Theory of the Grating Coupler	149
5.5.1	Description of the Incident Beam	150
5.5.2	Response of the Structure to a Plane Wave	151
5.5.3	Response of the Structure to a Limited Beam	153
5.5.4	Determination of the Coupling Coefficient	154
5.5.5	Application to a Limited Incident Beam	155
	References	156
6.	<i>Experimental Verifications and Applications of the Theory</i> By D. Maystre, M. Nevière and R. Petit (With 105 Figures)	159
6.1	Experimental Checking of Theoretical Results	159
6.1.1	Generalities	159
6.1.2	Microwave Region	160
6.1.3	On the Determination of Groove Geometry and of the Refractive Index	160
6.1.4	Infrared	164
6.1.5	Visible Region	165
6.1.6	Near and Vacuum UV	170

6.1.7	XUV Domain	171
6.1.8	X-Ray Domain	172
6.2	Systematic Study of the Efficiency of Perfectly Conducting Gratings .	173
6.2.1	Systematic Study of Echelette Gratings in -1 Order Littrow Mount	174
6.2.2	An Equivalence Rule Between Ruled, Holographic, and Lamellar Gratings	181
6.2.3	Systematic Study of the Efficiency of Holographic Gratings in -1 Order Littrow Mount	184
6.2.4	Systematic Study of the Efficiency of Symmetrical Lamellar Gratings in -1 Order Littrow Mount	188
6.2.5	Influence of the Apex Angle	190
6.2.6	Influence of a Departure from Littrow	191
6.2.7	Higher Order Use of Gratings	194
6.3	Finite Conductivity Gratings	198
6.3.1	General Rules	198
6.3.2	Typical Efficiency Curves in the Visible Region	201
6.3.3	Influence of Dielectric Overcoatings in Vacuum UV	202
6.3.4	The Use of Gratings in XUV and X-Ray Regions ($\lambda < 1000 \text{ \AA}$)	205
6.3.5	Conical Diffraction Mountings	209
6.4	Some Particular Applications	212
6.4.1	Simultaneous Blazing in Both Polarizations	212
6.4.2	Spectrometers with Constant Efficiency	213
6.4.3	Grating Bandpass Filter	214
6.4.4	Reflection Grating Polarizer for the Infrared	216
6.4.5	Transmission Gratings as Masks in Photolithography	216
6.4.6	Gratings Used as Beam Sampling Mirrors for High Power Lasers	218
6.4.7	Gratings as Wavelength Selectors in Tunable Lasers	220
6.4.8	Transmission Dielectric Gratings used as Color Filters	221
	Concluding Remarks	223
	References	223
7.	<i>Theory of Crossed Gratings</i>	
	By R.C. McPhedran, G.H. Derrick, and L.C. Botten (With 20 Figures)	227
7.1	Overview	227
7.2	The Bigrating Equation and Rayleigh Expansions	228
7.3	Inductive Grids	232
7.3.1	Grids with Rectangular Apertures	233
7.3.2	Numerical Tests and Applications	236
7.3.3	Inductive Grids with Circular Apertures	239

7.4 Capacitive and Other Grid Geometries	242
7.4.1 High-Pass Filters	243
7.4.2 Low-Pass Filters	243
7.4.3 Bandpass Filters	243
7.4.4 Bandstop Filters	244
7.5 Spatially Separated Grids or Gratings	244
7.5.1 The Crossed Lamellar Transmission Grating	245
7.5.2 The Double Grating	247
7.5.3 Symmetry Properties of Double Gratings	251
7.5.4 Multielement Grating Interference Filters	255
7.6 Finitely Conducting Bigratings	258
7.6.1 A Short Description of the Method	258
7.6.2 The Coordinate Transformation	259
7.6.3 Integral Equation Form	262
7.6.4 Iterative Solution of the Integral Equations	266
7.6.5 Total Absorption of Unpolarized Monochromatic Light	267
7.6.6 Reduction of Metallic Reflectivity: Plasmons and Moth-Eyes ...	269
7.6.7 Equivalence Formulae Linking Crossed and Classical Gratings ..	271
7.6.8 Coated Bigratings	273
References	275
<i>Additional References with Titles</i>	277
<i>Subject Index</i>	281