

Mesh Generation

Application to Finite Elements

Second Edition

Pascal Jean Frey
Paul-Louis George

ISTE

 WILEY

Contents

Introduction	11
Symbols and Notations	17
1 General Definitions	19
1.1 Covering-up and triangulation	20
1.2 Mesh, mesh element, finite element mesh	24
1.3 Mesh data structures	31
1.4 Control space and neighborhood space	39
1.5 Mesh quality and mesh optimality	41
2 Basic Structures and Algorithms	45
2.1 Why use data structures?	46
2.2 Elementary structures	48
2.3 Basic notions about complexity	53
2.4 Sorting and searching	56
2.5 One-dimensional data structures	62
2.6 Two and three-dimensional data structures	69
2.7 Topological data structures	74
2.8 Robustness	77
2.9 Optimality of an implementation	80
2.10 Examples of generic algorithms	83
3 A Comprehensive Survey of Mesh Generation Methods	95
3.1 Classes of methods	96
3.2 Structured mesh generators	97
3.2.1 Algebraic interpolation methods	98
3.2.2 PDE-based methods	100
3.2.3 Multiblock method	101
3.2.4 Product method (topology-based method)	103
3.3 Unstructured mesh generators	104
3.3.1 Spatial decomposition methods	106
3.3.2 Advancing-front method	109
3.3.3 Delaunay technique	114
3.3.4 Tentative comparison of the three classical methods	117

3.3.5	Other methods	119
3.4	Surface meshing	122
3.4.1	Mesh generation via a parametric space	122
3.4.2	Implicit surface triangulation	124
3.4.3	Direct surface meshing	125
3.4.4	Surface remeshing	126
3.5	Mesh adaptation	128
3.6	Parallel unstructured meshing	129
4	Algebraic, PDE and Multiblock Methods	133
4.1	Algebraic methods	133
4.1.1	Trivial mapping functions	134
4.1.2	Quadrilateral or triangular analogy	135
4.1.3	Surface meshing	143
4.1.4	Hexahedral, pentahedral or tetrahedral analogy	144
4.1.5	Other algebraic methods and alternative methods	147
4.2	PDE-based methods	149
4.2.1	Basic ideas	149
4.2.2	Surface meshing and complex shapes	153
4.3	Multiblock method	153
4.3.1	Basic ideas	153
4.3.2	Partitioning the domain	154
4.3.3	Computational issues and application examples	155
5	Quadtree-octree Based Methods	163
5.1	Overview of spatial decomposition methods	164
5.2	Classical tree-based mesh generation	171
5.3	Governed tree-based method	189
5.4	Other approaches	192
5.5	Extensions	195
6	Advancing-front Technique for Mesh Generation	201
6.1	A classical advancing-front technique	203
6.2	Governed advancing-front method	220
6.3	Application examples	225
6.4	Combined approaches	227
6.5	Extensions	230
7	Delaunay-based Mesh Generation Methods	235
7.1	Voronoi diagram and Delaunay triangulation	236
7.2	Constrained triangulation	245
7.2.1	Maintaining a constrained entity	245
7.2.2	Enforcing a constraint	246
7.3	Classical Delaunay meshing	250
7.3.1	Simplified Delaunay type triangulation method	252
7.3.2	Boundary integrity and domain identification	254
7.3.3	Field point creation	255

7.3.4	Optimization	257
7.3.5	Practical issues	258
7.3.6	Application examples	259
7.4	Other methods	261
7.4.1	Point insertion methods	262
7.4.2	Field point creation	262
7.4.3	Boundary enforcement	263
7.5	Isotropic governed Delaunay meshing	264
7.6	Extensions	267
7.6.1	Weighted Delaunay triangulation	268
7.6.2	Anisotropic Delaunay meshing	268
7.6.3	Surface meshing	273
8	Other Types of Mesh Generation Methods	275
8.1	Product method	276
8.2	Grid or pattern-based methods	280
8.3	Optimization-based method	283
8.4	Quads by means of triangle combination	290
8.5	Quads by means of a direct method	296
8.6	Hex meshing	298
8.7	Miscellaneous'	300
9	Delaunay Admissibility, Medial Axis and Applications	303
9.1	Delaunay-admissible set of segments in \mathbb{R}^2	304
9.2	Delaunay-admissible set of segments in \mathbb{R}^3	310
9.3	Delaunay-admissible set of triangular faces	312
9.4	Medial axis	318
9.5	Mid-surface	325
9.6	Applications	326
10	Quadratic Forms and Metrics	331
10.1	Bilinear and quadratic forms	332
10.2	Distances and lengths	337
10.3	Metric-based operations	342
10.4	Metric construction	350
10.4.1	Parametric surface meshing	351
10.4.2	Finite element simulation with error control	352
11	Differential Geometry	361
11.1	Metric properties of curves and arcs	362
11.2	Metric properties of a surface	377
11.3	Computational issues about surfaces	387
11.4	Non-linear problems	392

12 Curve Modeling	395
12.1 Interpolation and smoothing techniques	397
12.2 Lagrange and Hermite interpolation	400
12.3 Explicit construction of a composite curve	404
12.4 Control polygon based methods	406
12.5 Bézier curves	409
12.6 From composite curves to B-splines	414
12.7 Rational curves	423
12.8 Curve definitions and numerical issues	427
12.9 Towards a “pragmatic” curve definition?	431
13 Surface Modeling	435
13.1 Specific surfaces	436
13.2 Interpolation-based surfaces	437
13.3 Tensor product and control polyhedron	442
13.4 Triangular patches and Bézier triangles	446
13.5 Other types of patches	450
13.6 Composite surfaces	452
13.7 Explicit construction of a composite surface	457
14 Curve Meshing	463
14.1 Meshing a segment	464
14.2 Meshing a parametric curve	471
14.3 Curve meshing using a discrete definition	483
14.4 Re-meshing algorithm	486
14.5 Curves in \mathbb{R}^3	488
15 Surface Meshing and Re-meshing	491
15.1 Curve meshing (curve member of a surface)	492
15.2 First steps in surface meshing	493
15.3 A single patch	503
15.4 Multi-patches surface (patch-dependent)	513
15.5 Multi-patches surface (patch-independent)	515
15.6 Ill-defined multi-patches surface	517
15.7 Molecular surfaces	519
15.8 Surface reconstruction	521
15.9 Discrete surface (re-meshing process)	523
16 Meshing Implicit Curves and Surfaces	527
16.1 Review of implicit functions	528
16.2 Implicit function and meshing	533
16.3 Implicit curve meshing	537
16.4 Implicit surface meshing	545
16.5 Extensions	556

17 Mesh Modifications	559
17.1 Mesh (geometric) modifications	559
17.2 Merging two meshes	567
17.3 Node creation and node labeling	573
17.4 Renumbering issues	577
17.5 Miscellaneous	587
18 Mesh Optimization	591
18.1 About element measurement	592
18.2 Mesh quality (classical case)	596
18.3 Mesh quality (isotropic and anisotropic case)	602
18.4 Tools for mesh optimization	606
18.5 Strategies for mesh optimization	617
18.6 Computational issues	618
18.7 Application examples	619
19 Surface Mesh Optimization	623
19.1 Quality measures	624
19.2 Discrete evaluation of surface properties	632
19.3 Constructing a geometric support	643
19.4 Optimization operators	645
19.5 Optimization methods	654
19.6 Application examples	656
20 A Touch of Finite Elements	663
20.1 Introduction to a finite element style computation	664
20.2 Definition and first examples of finite elements	668
20.3 Error estimation and convergence	672
20.4 Stiffness matrix and right-hand side	677
20.5 A few examples of popular finite elements	690
21 Mesh Adaptation and H-methods	693
21.1 Control space (background mesh)	694
21.2 Adaptation by local modifications	701
21.3 Global isotropic adaptation method	708
21.4 Global anisotropic adaptation method	716
21.5 Adaptation	722
21.5.1 General framework of a local adaptation method	722
21.5.2 General framework of a global adaptation method	724
21.6 Application examples	727
22 Mesh Adaptation and P or H_p-methods	735
22.1 P^2 mesh	736
22.2 P -compatibility	742
22.3 Construction of P^2 elements	747
22.4 Elements of higher degree	750
22.5 P -methods and h_p -methods	751

23 Moving or Deformable Meshing Techniques	753
23.1 Rigid body motion	754
23.2 ALE methods	758
23.3 Mesh deformation	764
23.4 Interface tracking	767
24 Parallel Computing and Meshing Issues	775
24.1 Partition of a domain	776
24.2 Parallel meshing process	790
24.3 Parallel meshing techniques	792
Bibliography	799
Index	843