

# CAUSALITY

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## Models, Reasoning, and Inference

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**Epilogue The Art and Science of Cause and Effect**

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