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Multivariate Geostatistics

An Introduction with Applications

Third, completely revised edition

with 117 Figures and 7 Tables



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