

Advances in Distribution Theory, Order Statistics, and Inference

N. Balakrishnan
Enrique Castillo
José María Sarabia
Editors

Birkhäuser
Boston • Basel • Berlin

Contents

Preface	xix
Barry C. Arnold: Career and Accomplishments	xxi
Publications	xxiii
Contributors	xli
List of Tables	xlvi
List of Figures	li

PART I: DISCRETE DISTRIBUTIONS AND APPLICATIONS

1 Stochastic Comparisons of Bernoulli Sums and Binomial Random Variables	3
<i>P. J. Boland and H. Singh</i>	
1.1 Introduction	3
1.2 Stochastic Orders for Random Variables	5
1.3 Stochastic Order Comparisons for Sums of Bernoulli Random Variables	6
1.4 Graphical Insight for Two-Dimensional Stochastic Comparisons	8
References	11
2 Stopped Compound Poisson Process and Related Distributions	13
<i>C. Lefèvre</i>	
2.1 Introduction	13
2.2 The Boundary Is Linear	15
2.3 The Boundary Is of Renewal Type	18
2.4 The Boundary Is Any Deterministic Function	19
2.5 A Higher Deterministic Boundary	22
References	25

3	Constructions of Discrete Bivariate Distributions	29
	<i>C. D. Lai</i>	
3.1	Introduction	29
3.2	Mixing and Compounding	30
3.2.1	Mixing	30
3.2.2	Compounding	31
3.3	Trivariate Reduction	32
3.4	One Conditional and One Marginal Given	33
3.5	Conditionally Specified Method	34
3.6	Construction of Discrete Bivariate Distributions with Given Marginals and Correlation	35
3.6.1	Discrete Fréchet bounds	35
3.6.2	Probability functions of Fréchet bounds	35
3.6.3	Construction of bivariate distributions	36
3.6.4	Construction of bivariate Poisson distributions	37
3.7	Sums and Limits of Bernoulli Trials	38
3.7.1	The bivariate Bernoulli distribution	38
3.7.2	Construction of bivariate Bernoulli distributions	38
3.8	Sampling from Urn Models	38
3.9	Clustering (Bivariate Distributions of Order k)	40
3.9.1	Preliminary	40
3.9.2	Bivariate distributions of order k	40
3.10	Construction of Finite Bivariate Distributions via Extreme Points of Convex Sets	41
3.10.1	Finding extreme points	43
3.11	Generalized Distributions	43
3.11.1	Generalized bivariate distributions	44
3.11.2	Generalized bivariate Poisson distributions	44
3.11.3	Generalized bivariate general binomial distributions	45
3.12	Canonical Correlation Coefficients and Semigroups	45
3.12.1	Diagonal expansion	45
3.12.2	Canonical correlation coefficients and positive definite sequence	46
3.12.3	Moment sequence and canonical correlation coefficient	46
3.12.4	Constructions of bivariate distributions via canonical sequences	46
3.13	Bivariate Distributions from Accident Models	47
3.13.1	The Poisson-Poisson, Poisson-binomial, and Poisson-Bernoulli methods	47
3.13.2	Negative binomial-Poisson and negative binomial-Bernoulli models	48
3.14	Bivariate Distributions Generated from Weight Functions	48
3.15	Marginal Transformation Method	48

3.16	Truncation Methods	49
3.17	Construction of Positively Dependent Discrete Bivariate Distributions	50
3.17.1	Positive quadrant dependent distributions	50
3.17.2	Positive regression dependent distributions	51
	References	51

PART II: CONTINUOUS DISTRIBUTIONS AND APPLICATIONS

4	The Normal-Laplace Distribution and Its Relatives	61
	<i>W. J. Reed</i>	
4.1	Introduction	61
4.2	The Normal-Laplace Distribution	63
4.3	Related Distributions	68
4.3.1	The double Pareto-lognormal distribution	68
4.3.2	The generalized normal-Laplace distribution	68
4.4	A Lévy Motion Based on the GNL Distribution	70
4.4.1	Option pricing for assets with logarithmic prices following Brownian-Laplace motion	70
4.5	Estimation for ML and GNL Distributions	73
	References	73
5	Some Observations on a Simple Means of Generating Skew Distributions	75
	<i>A. Pewsey</i>	
5.1	Introduction	75
5.2	Flexibility and Limitations of the Construct	76
5.3	Inference	79
5.3.1	General considerations	79
5.3.2	Score equations for any $S_{fG}(\xi, \eta, \lambda)$ class	80
5.3.3	Observed information matrix for any $S_{fG}(\xi, \eta, \lambda)$ class	81
	References	83
6	Bivariate Distributions Based on the Generalized Three-Parameter Beta Distribution	85
	<i>J. M. Sarabia and E. Castillo</i>	
6.1	Introduction	85
6.2	The Generalized Three-parameter Best Distribution	87
6.2.1	Relationships with other distributions and extensions	88
6.3	Models with Generalized Three-Parameter Beta Marginals	89
6.3.1	Model based on the Dirichlet distribution	90
6.3.2	Model based on the Sarmanov-Lee distribution	91

6.4 The Generalized Three-Parameter Beta Conditionals Distribution	92
6.4.1 The generalized beta conditionals distribution with $\lambda_i(\cdot)$ constant	93
6.4.2 The generalized beta conditionals distribution with constant $a_i(\cdot)$ and $b_i(\cdot)$	97
6.4.3 Dependence conditions	100
6.5 Bivariate Distributions with Gauss Hypergeometric Conditionals	102
6.5.1 A flexible model	103
6.6 Other Bivariate Distributions with Specified Conditionals	104
6.7 Applications to Bayesian Inference	105
6.8 Conditional Survival Models	106
6.9 Multivariate Extensions	107
References	108
7 A Kotz-Type Distribution for Multivariate Statistical Inference	111
<i>D. N. Naik and K. Plungpongpan</i>	
7.1 Introduction	111
7.1.1 Moments and other properties	113
7.1.2 Marginal and conditional distributions	114
7.2 An Algorithm for Simulation	114
7.3 Estimation of Parameters	117
7.3.1 Generalized spatial median (GSM)	118
7.3.2 Computation of GSM and $\hat{\Sigma}$	118
7.3.3 The asymptotic distribution of GSM	119
7.4 An Example	121
References	122
8 Range of Correlation Matrices for Dependent Random Variables with Given Marginal Distributions	125
<i>H. Joe</i>	
8.1 Introduction	125
8.2 Known Results on a Range of Correlations	127
8.3 Conditional Approach	128
8.3.1 Multivariate normal and partial correlations	128
8.3.2 General case	130
8.4 Characterization of F for $S_d(F) = S_d^*$	132
8.4.1 $d = 3$	133
8.4.2 $d > 3$	136
8.5 Discussion	141
References	141

9 Multifractional Probabilistic Laws **143**

M. D. Ruiz-Medina and J. M. Angulo

- 9.1 Introduction 143
- 9.2 Preliminaries 144
- 9.3 Fractional Differential Characterization 147
- 9.4 Multifractional Versions 148
- 9.5 Fractional and Multifractional Moment Laws 150
 - 9.5.1 Multifractal moment laws 151
- 9.6 Conclusion 152
- References 152

PART III: ORDER STATISTICS AND APPLICATIONS

10 Topics in the History of Order Statistics **157**

H. A. David

- 10.1 Introduction 157
- 10.2 Early Measures of Location 158
- 10.3 Distribution Theory 162
- 10.4 Extreme-Value Theory 163
- 10.5 Estimation of Location and Scale Parameters by Linear Functions of Order Statistics 166
- 10.6 Tables 167
- References 168

11 Order Statistics from Independent Exponential Random Variables and the Sum of the Top Order Statistics **173**

H. N. Nagaraja

- 11.1 Introduction 173
- 11.2 Distributional Representation and Basic Applications 174
 - 11.2.1 Remarks 175
 - 11.2.2 Applications 176
- 11.3 Sum of the Top Order Statistics 178
 - 11.3.1 The IID case 179
 - 11.3.2 The non-IID case 180
 - 11.3.3 The IID case vs. the INID case 183
- References 184

12 Fisher Information and Tukey's Linear Sensitivity Measure Based on Ordered Ranked Set Samples	187
<i>N. Balakrishnan and T. Li</i>	
12.1 Introduction	188
12.2 Maximum Likelihood Estimation Based on the ORSS	189
12.2.1 Logistic distribution	194
12.2.2 Normal distribution	195
12.2.3 One-parameter exponential distribution	196
12.2.4 Conclusions	197
12.3 Tukey's Linear Sensitivity Measure Based on ORSS	198
Appendix	203
References	204
13 Information Measures for Pareto Distributions and Order Statistics	207
<i>M. Asadi, N. Ebrahimi, G. G. Hamedani, and E. S. Soofi</i>	
13.1 Introduction	207
13.2 Information Measures	208
13.2.1 Shannon entropy	209
13.2.2 Rényi information measures	210
13.2.3 Dynamic information	210
13.2.4 Maximum entropy and maximum dynamic entropy	212
13.3 Information Properties of Pareto Distributions	213
13.3.1 Characterizations of generalized Pareto	214
13.3.2 ME, MED, and $MDE\alpha$ characterizations of Pareto	217
13.4 Information Properties of Order Statistics	217
References	221
14 Confidence Coefficients of Interpolated Nonparametric Sign Intervals for Medians Under No or Weak Shape Assumptions	225
<i>O. Guilbaud</i>	
14.1 Introduction	225
14.2 Confidence Coefficient Under No Shape Assumption	227
14.3 Confidence Coefficient Under Symmetry	228
14.4 Confidence Coefficient Under Symmetry and Unimodality	229
14.5 Domination Relations Among Interval Estimators	230
14.6 Nondominated Interval Estimators and Available Confidence Coefficients	231

14.7 Concluding Comments and Additional Results	233
Appendices	234
References	237
15 Small Sample Asymptotics for Higher-Order Spacings	239
<i>R. Gatto and S. R. Jammalamadaka</i>	
15.1 Introduction	239
15.2 Tests Based on Higher-Order Spacings	241
15.3 Tests Based on Higher-Order Spacing-Frequencies	245
15.4 Conclusion	250
References	250
16 Best Bounds on Expectations of L-Statistics from Bounded Samples	253
<i>T. Rychlik</i>	
16.1 Introduction	253
16.2 General Results	254
16.3 Special Cases	259
References	262
PART IV: RELIABILITY AND APPLICATIONS	
17 The Failure Rates of Mixtures	267
<i>H. W. Block</i>	
17.1 Introduction	267
17.2 Notation	268
17.3 Examples	269
17.4 Asymptotics	270
17.5 Mixtures of Distributions with Linear Failure Rates	271
17.6 Mixtures of Standard Reliability Distributions	272
17.7 Preservation Under Mixtures	273
17.8 Analytic Tools for Determining the Shape of Mixtures	273
17.9 Coherent Systems	274
17.10 Summary of Overall Shape	275
References	275
18 Characterizations of the Relative Behavior of Two Systems via Properties of Their Signature Vectors	279
<i>H. Block, M. R. Dugas, and F. J. Samaniego</i>	
18.1 Introduction	279
18.2 Background Results for the Comparison of System Life	281

18.3 New Signature Conditions and Associated System Behavior	284
18.4 Practical Implications	286
References	289
19 Systems with Exchangeable Components and Gumbel Exponential Distribution	291
<i>J. Navarro, J. M. Ruiz, and C. J. Sandoval</i>	
19.1 Introduction	291
19.2 General Properties	292
19.3 Reliability and Moments	294
19.4 Aging Measures	298
19.5 Stochastic Orders and Classes	299
19.6 Parameter Estimation	302
19.7 Systems with n Exchangeable Components	303
References	305
20 Estimating the Mean of Exponential Distribution from Step-Stress Life Test Data	307
<i>Z. Chen, J. Mi, and Y. Y. Zhou</i>	
20.1 Introduction	307
20.2 Type I Censored Data	309
20.3 Grouped Data	314
20.4 Type II Censored Data	317
20.5 Simulation Study	324
References	325
21 Random Stress-Dependent Strength Models Through Exponential Conditionals Distributions	327
<i>A. SenGupta</i>	
21.1 Introduction	327
21.2 Bivariate Exponential Conditionals Distribution	328
21.3 Properites of BCE	330
21.3.1 Dependency properties of BCE	331
21.4 Model Representations in ALT	332
21.5 Statistical Inference under Normal Stress	333
21.5.1 Estimation of α and β	334
21.5.2 Asymptotic inference for θ_0	335
21.6 Unconditional Reliability Function and Measure	336
21.7 Conclusions	337
References	338

PART V: INFERENCE

22 Some New Methods for Local Sensitivity Analysis in Statistics	343
<i>E. Castillo, C. Castillo, A. S. Hadi, and J. M. Sarabia</i>	
22.1 Introduction and Motivation	343
22.2 Sensitivities of the Objective Function	344
22.3 Applications to Regression	346
22.3.1 Least-squares regression	346
22.3.2 Minimax regression	347
22.3.3 Mixed least-squares and minimax regression	348
22.3.4 Example: Simulated data	348
22.4 The Maximum Likelihood Function	350
22.4.1 Local sensitivities	351
22.4.2 Examples: The gamma and beta families	351
22.5 Ordered and Data Constrained Parameters	351
22.6 The Method of Moments Estimates	354
22.6.1 Local sensitivities	354
22.6.2 Example 1: The gamma family	355
22.6.3 Example 2: The beta family	356
22.7 Conclusions	358
References	359
23 <i>t</i>-Tests with Models Close to the Normal Distribution	363
<i>A. García-Pérez</i>	
23.1 Introduction	363
23.2 Preliminaries	364
23.2.1 Influence functions of p_n^F and k_n^F	366
23.2.2 Von Mises expansions of p_n^F and k_n^F	367
23.2.3 Von Mises approximations of p_n^F and k_n^F with a model F close to the normal distribution	368
23.3 Von Mises Approximations for <i>t</i> -Tests	369
23.4 Saddlepoint Approximations for <i>t</i> -Tests	373
References	378
24 Computational Aspect of the Chi-Square Goodness-of-Fit Test Applications	381
<i>M. Divinsky</i>	
24.1 Introduction	381
24.2 On the Chi-Square Test Application	382
24.3 An Actual Data Set	383
24.4 Modeled Sample of the Generated Values	385

24.5	Conclusions	386
	References	387
25	An Objective Bayesian Procedure for Variable Selection in Regression	389
	<i>F. J. Girón, E. Moreno, and M. L. Martínez</i>	
25.1	Introduction	389
25.2	Intrinsic Priors for Variable Selection	391
25.3	Bayes Factors and Model Posterior Probabilities	393
25.4	Relation with the R^2 and Other Classical Criterion for Model Selection	394
25.5	Examples	397
25.5.1	Simulation study	397
25.5.2	Hald's data	398
25.5.3	Prostate cancer data	399
25.6	Conclusions	401
	References	402
26	On Bayesian and Decision-Theoretic Approaches to Statistical Prediction	405
	<i>T. K. Nayak and A. El-Baz</i>	
26.1	Introduction	405
26.2	Bayesian Prediction	407
26.3	Admissible Predictors	410
	References	414
27	Phi-Divergence-Type Test for Positive Dependence Alternatives in $2 \times k$ Contingency Tables	417
	<i>L. Pardo and M. L. Menéndez</i>	
27.1	Introduction	417
27.2	Phi-Divergence Test Statistics	419
27.3	Asymptotic Distribution of the ϕ -Divergence Test Statistics	425
	References	430
28	Dimension Reduction in Multivariate Time Series	433
	<i>D. Peña and P. Poncela</i>	
28.1	Introduction	433
28.2	Models for Dimension Reduction	435
28.2.1	Principal components	435
28.2.2	The Box and Tiao canonical analysis	436
28.2.3	Reduced rank models	438

28.2.4 The scalar components models	439
28.2.5 Dynamic factor models	440
28.2.6 State space models	441
28.2.7 Some conclusions	442
28.3 Dimension Reduction Tests	443
28.3.1 A test for zero canonical correlation coefficients	443
28.3.2 A nonstandard test for canonical correlations	445
28.3.3 A canonical correlation test for factor models	448
28.4 Real Data Analysis	449
28.5 Concluding Remarks	455
References	456
29 The Hat Problem and Some Variations	459
<i>W. Guo, S. Kasala, M. B. Rao, and B. Tucker</i>	
29.1 Introduction	459
29.2 Hamming Codes	461
29.3 Three Team Mates and Three Colors	464
29.4 Three Team Mates and m Colors	466
29.5 An Upper Bound for the Winning Probability	468
29.6 General Distribution	471
29.7 Other Variations	478
29.8 Some Open Problems	478
29.9 The Yeast Genome Problem	478
References	479
Index	481