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# Fundamentals of Robotic Mechanical Systems

Theory, Methods, and Algorithms

Second Edition



Springer

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