

AN INTRODUCTION TO  
**Error Analysis**

THE STUDY OF UNCERTAINTIES  
IN PHYSICAL MEASUREMENTS

SECOND EDITION

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