

# Optimization Methods in Finance

Second Edition

GÉRARD CORNUÉJOLS

Carnegie Mellon University, Pennsylvania

JAVIER PEÑA

Carnegie Mellon University, Pennsylvania

REHA TÛTÛNCÛ

SECOR Asset Management



**CAMBRIDGE**  
UNIVERSITY PRESS

# Contents

*Preface*

*page xi*

<b>Part I</b>	<b>Introduction</b>	<b>1</b>
<b>1</b>	<b>Overview of Optimization Models</b>	<b>3</b>
	1.1 Types of Optimization Models	4
	1.2 Solution to Optimization Problems	7
	1.3 Financial Optimization Models	8
	1.4 Notes	10
<b>2</b>	<b>Linear Programming: Theory and Algorithms</b>	<b>11</b>
	2.1 Linear Programming	11
	2.2 Graphical Interpretation of a Two-Variable Example	15
	2.3 Numerical Linear Programming Solvers	16
	2.4 Sensitivity Analysis	17
	2.5 *Duality	20
	2.6 *Optimality Conditions	23
	2.7 *Algorithms for Linear Programming	24
	2.8 Notes	30
	2.9 Exercises	31
<b>3</b>	<b>Linear Programming Models: Asset–Liability Management</b>	<b>35</b>
	3.1 Dedication	35
	3.2 Sensitivity Analysis	38
	3.3 Immunization	38
	3.4 Some Practical Details about Bonds	41
	3.5 Other Cash Flow Problems	44
	3.6 Exercises	47
	3.7 Case Study	51
<b>4</b>	<b>Linear Programming Models: Arbitrage and Asset Pricing</b>	<b>53</b>
	4.1 Arbitrage Detection in the Foreign Exchange Market	53
	4.2 The Fundamental Theorem of Asset Pricing	55
	4.3 One-Period Binomial Pricing Model	56

---

4.4	Static Arbitrage Bounds	59
4.5	Tax Clientele Effects in Bond Portfolio Management	63
4.6	Notes	65
4.7	Exercises	65
<b>Part II</b>	<b>Single-Period Models</b>	<b>69</b>
<b>5</b>	<b>Quadratic Programming: Theory and Algorithms</b>	<b>71</b>
5.1	Quadratic Programming	71
5.2	Numerical Quadratic Programming Solvers	74
5.3	Sensitivity Analysis	75
5.4	*Duality and Optimality Conditions	76
5.5	*Algorithms	81
5.6	Applications to Machine Learning	84
5.7	Exercises	87
<b>6</b>	<b>Quadratic Programming Models: Mean–Variance Optimization</b>	<b>90</b>
6.1	Portfolio Return	90
6.2	Markowitz Mean–Variance (Basic Model)	91
6.3	Analytical Solutions to Basic Mean–Variance Models	95
6.4	More General Mean–Variance Models	99
6.5	Portfolio Management Relative to a Benchmark	103
6.6	Estimation of Inputs to Mean–Variance Models	106
6.7	Performance Analysis	112
6.8	Notes	115
6.9	Exercises	115
6.10	Case Studies	121
<b>7</b>	<b>Sensitivity of Mean–Variance Models to Input Estimation</b>	<b>124</b>
7.1	Black–Litterman Model	126
7.2	Shrinkage Estimation	129
7.3	Resampled Efficiency	131
7.4	Robust Optimization	132
7.5	Other Diversification Approaches	133
7.6	Exercises	135
<b>8</b>	<b>Mixed Integer Programming: Theory and Algorithms</b>	<b>140</b>
8.1	Mixed Integer Programming	140
8.2	Numerical Mixed Integer Programming Solvers	143
8.3	Relaxations and Duality	145
8.4	Algorithms for Solving Mixed Integer Programs	150
8.5	Exercises	157

<b>9</b>	<b>Mixed Integer Programming Models: Portfolios with Combinatorial Constraints</b>	161
	9.1 Combinatorial Auctions	161
	9.2 The Lockbox Problem	163
	9.3 Constructing an Index Fund	165
	9.4 Cardinality Constraints	167
	9.5 Minimum Position Constraints	168
	9.6 Risk-Parity Portfolios and Clustering	169
	9.7 Exercises	169
	9.8 Case Study	171
<b>10</b>	<b>Stochastic Programming: Theory and Algorithms</b>	173
	10.1 Examples of Stochastic Optimization Models	173
	10.2 Two-Stage Stochastic Optimization	174
	10.3 Linear Two-Stage Stochastic Programming	175
	10.4 Scenario Optimization	176
	10.5 *The L-Shaped Method	177
	10.6 Exercises	179
<b>11</b>	<b>Stochastic Programming Models: Risk Measures</b>	181
	11.1 Risk Measures	181
	11.2 A Key Property of CVaR	185
	11.3 Portfolio Optimization with CVaR	186
	11.4 Notes	190
	11.5 Exercises	190
<b>Part III</b>	<b>Multi-Period Models</b>	195
<b>12</b>	<b>Multi-Period Models: Simple Examples</b>	197
	12.1 The Kelly Criterion	197
	12.2 Dynamic Portfolio Optimization	198
	12.3 Execution Costs	201
	12.4 Exercises	209
<b>13</b>	<b>Dynamic Programming: Theory and Algorithms</b>	212
	13.1 Some Examples	212
	13.2 Model of a Sequential System (Deterministic Case)	214
	13.3 Bellman's Principle of Optimality	215
	13.4 Linear-Quadratic Regulator	216
	13.5 Sequential Decision Problem with Infinite Horizon	218
	13.6 Linear-Quadratic Regulator with Infinite Horizon	219
	13.7 Model of Sequential System (Stochastic Case)	221
	13.8 Notes	222
	13.9 Exercises	222

---

<b>14</b>	<b>Dynamic Programming Models: Multi-Period Portfolio Optimization</b>	<b>225</b>
14.1	Utility of Terminal Wealth	225
14.2	Optimal Consumption and Investment	227
14.3	Dynamic Trading with Predictable Returns and Transaction Costs	228
14.4	Dynamic Portfolio Optimization with Taxes	230
14.5	Exercises	234
<b>15</b>	<b>Dynamic Programming Models: the Binomial Pricing Model</b>	<b>238</b>
15.1	Binomial Lattice Model	238
15.2	Option Pricing	238
15.3	Option Pricing in Continuous Time	244
15.4	Specifying the Model Parameters	245
15.5	Exercises	246
<b>16</b>	<b>Multi-Stage Stochastic Programming</b>	<b>248</b>
16.1	Multi-Stage Stochastic Programming	248
16.2	Scenario Optimization	250
16.3	Scenario Generation	255
16.4	Exercises	259
<b>17</b>	<b>Stochastic Programming Models: Asset–Liability Management</b>	<b>262</b>
17.1	Asset–Liability Management	262
17.2	The Case of an Insurance Company	263
17.3	Option Pricing via Stochastic Programming	265
17.4	Synthetic Options	270
17.5	Exercises	273
<b>Part IV</b>	<b>Other Optimization Techniques</b>	<b>275</b>
<b>18</b>	<b>Conic Programming: Theory and Algorithms</b>	<b>277</b>
18.1	Conic Programming	277
18.2	Numerical Conic Programming Solvers	282
18.3	Duality and Optimality Conditions	282
18.4	Algorithms	284
18.5	Notes	287
18.6	Exercises	287
<b>19</b>	<b>Robust Optimization</b>	<b>289</b>
19.1	Uncertainty Sets	289
19.2	Different Flavors of Robustness	290
19.3	Techniques for Solving Robust Optimization Models	294
19.4	Some Robust Optimization Models in Finance	297
19.5	Notes	302
19.6	Exercises	302

---

---

<b>20</b>	<b>Nonlinear Programming: Theory and Algorithms</b>	<b>305</b>
20.1	Nonlinear Programming	305
20.2	Numerical Nonlinear Programming Solvers	306
20.3	Optimality Conditions	306
20.4	Algorithms	308
20.5	Estimating a Volatility Surface	315
20.6	Exercises	319
<b>Appendices</b>		<b>321</b>
<b>Appendix</b>	<b>Basic Mathematical Facts</b>	<b>323</b>
A.1	Matrices and Vectors	323
A.2	Convex Sets and Convex Functions	324
A.3	Calculus of Variations: the Euler Equation	325
<b>References</b>		<b>327</b>
<b>Index</b>		<b>334</b>