

An Economic Interpretation of
LINEAR
PROGRAMMING

Quirino Paris

Iowa State University Press / Ames

Contents

| | |
|--|-------------|
| Preface | xiii |
| 1 Linear Programming | 3 |
| 1.1 What Is Linear Programming? | 3 |
| 1.2 Why Another Linear Programming Book? | 5 |
| 1.3 Problems for Linear Programming | 5 |
| 1.4 A Profit-Maximizing Problem | 9 |
| 1.5 A First Encounter with Duality | 10 |
| 1.6 Profit Maximization in LP: The Primal Problem | 13 |
| 1.7 Free Goods | 14 |
| 1.8 Cost Minimization in LP: The Dual Problem | 15 |
| 1.9 Graphical Solution of LP Problems | 18 |
| 1.10 Additional Information: The Dual of the Dual | 20 |
| 1.11 Slack (Surplus) Variables and Their Meaning | 22 |
| 1.12 A General Linear Programming Formulation | 24 |
| 1.13 Three Crucial Assumptions | 25 |
| 2 Primal and Dual LP Problems | 27 |
| 2.1 The Structure of Duality | 27 |
| 2.2 Dual Pairs of LP Problems with Equality Constraints . . . | 29 |
| 2.3 Dual Problems with Equality Constraints and Free-Variables | 32 |
| 3 Setting Up LP Problems | 35 |
| 3.1 Preliminaries | 35 |
| 3.2 Problem 1: A Small Family Farm | 36 |
| 3.3 Problem 2: The Expanded Family Farm | 42 |
| 3.4 Problem 3: A Grape Farm | 46 |
| 3.5 Problem 4: The Expanded Grape Farm | 49 |
| 3.6 Problem 5: A Sequential Production | 51 |
| 3.7 Problem 6: A Horticultural Farm | 54 |
| 3.8 Problem 7: A Detailed Example | 57 |

| | | |
|----------|--|-----------|
| 4 | The Transportation Problem | 61 |
| 4.1 | Introduction | 61 |
| 4.2 | The Dual of the Transportation Problem | 63 |
| 4.3 | A Numerical Transportation Problem | 64 |
| 5 | Spaces and Bases | 67 |
| 5.1 | Introduction | 67 |
| 5.2 | Two Ways of Looking at a System of Constraints | 68 |
| 5.3 | A Primer on Vectors and Bases | 70 |
| 5.3.1 | Addition of Two Vectors | 71 |
| 5.3.2 | Subtraction of Two Vectors | 71 |
| 5.3.3 | Multiplication of a Vector by a Scalar | 73 |
| 5.3.4 | Extreme Points | 73 |
| 5.3.5 | Basis | 73 |
| 5.4 | Feasible Bases and Feasible Solutions | 77 |
| 5.5 | The Dual Definition of a Line or a Plane | 81 |
| 6 | Solving Systems of Equations | 83 |
| 6.1 | Introduction | 83 |
| 6.2 | The Gaussian Elimination Method | 84 |
| 6.3 | The Pivot Method | 85 |
| 6.3.1 | The Transformation Matrix | 86 |
| 6.3.2 | The Choice of a Pivot | 88 |
| 6.3.3 | Another Numerical Example | 89 |
| 6.4 | The Inverse of a Basis | 90 |
| 6.5 | A General View of the Pivot Method | 93 |
| 7 | The Primal Simplex Algorithm | 95 |
| 7.1 | Introduction to LP Algorithms | 95 |
| 7.2 | The Primal Simplex Algorithm | 97 |
| 7.2.1 | Ideas Underlying the Primal Simplex Method | 99 |
| 7.2.2 | The Marginal Rate of Technical Transformation . . | 106 |
| 7.2.3 | The Opportunity Cost of a Commodity | 109 |
| 7.2.4 | Recapitulation | 113 |
| 7.2.5 | Termination of the Primal Simplex Algorithm | 115 |
| 7.3 | Dual Variables and the Dual Interpretation of Opportunity Costs | 116 |
| 7.4 | Summary of the Primal Simplex Algorithm | 121 |

| | |
|---|------------|
| 8 The Dual Simplex Algorithm | 123 |
| 8.1 Introduction | 123 |
| 8.2 The Dual Simplex Algorithm | 126 |
| 8.3 Termination of the Dual Simplex Algorithm | 129 |
| 8.4 An Application of the Dual Simplex Algorithm | 130 |
| 8.5 Degenerate and Multiple Optimal Solutions | 132 |
| 9 Linear Programming and the Lagrangean Function | 137 |
| 9.1 The Lagrangean Method | 137 |
| 9.2 The Complementary Slackness Conditions | 140 |
| 9.3 Complementary Slackness at Work | 142 |
| 10 The Artificial Variable Algorithm | 145 |
| 10.1 Introduction | 145 |
| 10.2 Ideas Underlying the Artificial Variable Algorithm | 146 |
| 10.3 Termination of the Artificial Variable Algorithm | 155 |
| 10.4 Another Application of the Artificial Variable Algorithm | 155 |
| 11 The Artificial Constraint Algorithm | 159 |
| 11.1 Ideas Underlying the Artificial Constraint Algorithm | 159 |
| 11.2 Termination of the Artificial Constraint Algorithm | 163 |
| 12 The Diet Problem Revisited | 165 |
| 12.1 The Simplex Solution of Stigler's Problem | 165 |
| 12.2 The Optimal Diet from 77 Food Categories | 169 |
| 12.3 A Multiperiod Least-Cost Diet Problem | 169 |
| 13 Parametric Programming: Input Demand Functions | 179 |
| 13.1 Introduction | 179 |
| 13.2 Derivation of Input Demand Functions | 180 |
| 13.3 Connection with Linear Programming | 182 |
| 13.4 The Derived Demand for Input #1 | 185 |
| 13.4.1 Step 1: Increase in b_i (the Rybczynski Theorem) | 186 |
| 13.4.2 Step 1: Decrease in b_i | 190 |
| 13.4.3 Step 2: A Further Increase in b_i | 192 |
| 13.4.4 Step 2: A Further Decrease in b_i | 193 |
| 13.5 The Derived Demand for Input #2 | 196 |
| 13.5.1 Step 1: Increase and Decrease in b_2 | 197 |

| | |
|--|------------|
| 14 Parametric Programming: Output Supply Functions | 203 |
| 14.1 Derivation of Output Supply Functions | 203 |
| 14.2 The Supply Function for Output #2 | 205 |
| 14.2.1 The Opportunity Cost Revisited | 207 |
| 14.2.2 Increase in Price C_2 (the Stolper-Samuelson Theorem) | 211 |
| 14.2.3 Further Increase of C_2 | 215 |
| 14.2.4 Decrease in Price C_2 | 216 |
| 14.3 The Supply Function for Output #3 | 220 |
| 14.3.1 Increase of C_3 | 220 |
| 14.3.2 Decrease in C_3 | 222 |
| 14.4 Parametric Analysis of a Price Whose Activity Is Not in the Optimal Basis | 223 |
| 14.5 Symmetry in Parametric Programming | 224 |
| 15 Dealing with Multiple Optimal Solutions | 227 |
| 15.1 Introduction | 227 |
| 15.2 Choosing among Multiple Optimal Solutions | 229 |
| 15.3 Dual Multiple Optimal Solutions | 233 |
| 15.4 Problems with Multiple Primal and Dual Optimal Solutions | 235 |
| 15.5 Modeling to Generate Alternatives | 237 |
| 16 Solid Waste Management | 241 |
| 16.1 A Growing Problem | 241 |
| 16.2 A LP Model of Waste Management | 241 |
| 16.3 The Dual Problem | 245 |
| 16.4 Solution of the Waste Management Problem | 247 |
| 16.5 Parametric Analysis of Solid Waste Management | 250 |
| 17 The Choice of Techniques in a Farm Production Model | 255 |
| 17.1 Introduction | 255 |
| 17.2 A Numerical Farm Production Model | 258 |
| 17.3 The Derived Demands for Land and Labor | 268 |
| 18 Cattle Ranch Management | 271 |
| 18.1 A Static Model of Ranch Management | 271 |
| 18.2 A Multistage Model of Ranch Management | 282 |
| 19 The Measurement of Technical and Economic Efficiency | 287 |
| 19.1 The Notion of Relative Efficiency | 287 |
| 19.2 The Farrell Method | 290 |
| 19.3 Linear Programming and Efficiency Indexes | 291 |

| | |
|---|------------|
| 19.4 Technical and Economic Efficiency in a Group of Small Family Farms | 295 |
| 19.5 Efficiency in Multiproduct Firms | 300 |
| 19.6 An Example of Efficiency in Multiproduct Farms | 302 |
| 19.7 Efficiency and Returns to Scale | 303 |
| 20 Decentralized Economic Planning | 307 |
| 20.1 When Is Planning Necessary? | 307 |
| 20.2 Decentralized Economic Planning and LP | 308 ✓ |
| 20.3 The Western Scheme of Economic Planning | 310 |
| 20.4 The Russian Scheme of Economic Planning | 318 |
| 20.5 A Comparison of Planning Schemes | 323 |
| Bibliography | 325 |
| Glossary | 327 |
| Index | 333 |