FOURTH EDITION

Introduction to Random Signals and Applied Kalman Filtering

WITH MATLAB EXERCISES

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Rockwell Collins, Inc.

WILEY
John Wiley & Sons, Inc.
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