

SCHAUM'S OUTLINE OF
THEORY AND PROBLEMS
OF
LAGRANGIAN DYNAMICS

with a treatment of
**Euler's Equations of Motion,
Hamilton's Equations
and Hamilton's Principle**

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