

The (topo)logic of vagueness*

Brian Hill

16th January 2007

Abstract

Zeno's "dichotomy" paradox of the runner and the sorites paradox exhibit certain interesting similarities. Both of them involve a long series of steps, each of which seems legitimate, but which, taken together, apparently lead to an unacceptable conclusion. In this article, a particular interpretation of a common reply to Zeno's paradox is presented, which recognises that to defuse the paradox, it is necessary to assert that the number of stages that the runner has completed on Zeno's infinite sequence of times is not an appropriate measure of whether he finishes the race or not. Applying this style of reply to the sorites argument, one would reject the argument on the grounds of the inappropriateness of the number of hairs for reasoning about baldness. Such an attitude to the sorites argument implies a certain conception of the problem posed by vague terms, according to which the problem is to understand such relationships between terms as the appropriateness of one for reasoning about the other. Consequently, it poses a certain set of challenges to prospective theories of vagueness.

Keywords Vagueness, theories of, problem of; sorites paradox; Zeno's paradox; scale.

Zeno's "dichotomy" paradox argues that a runner in a race will never finish: before reaching the finish, he must get to the half-way point $p/2$ but once he has got to $p/2$, he must still reach the point half-way between $p/2$ and the finish, say $p/4$; but once he has reached $p/4$ • • • So, one might conclude, he never really finishes the race.

The sorites paradox argues that a man with a full head of hair may count as bald: for a man with 0 hairs counts as bald; and if a man with 0 hairs counts as bald, a man with 1 hair counts as bald; and if a man with 1 hair counts as bald, a

*Submitted to *Ratio*, January 2007.