

Series on Mathematical Economics and Game Theory

**Vol. 5**

# **FIXED POINTS AND ECONOMIC EQUILIBRIA**

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