

j

RESEARCH-^{in this} PRINCIPLES AND PRACTICE

DON T. PHILLIPS

DEPARTMENT OF INDUSTRIAL ENGINEERING
TEXAS A & M UNIVERSITY
COLLEGE STATION, TEXAS

i

A. RAVINDRAN

JAMES J. SOLBERG

SCHOOL OF INDUSTRIAL ENGINEERING
PURDUE UNIVERSITY
WEST LAFAYETTE, INDIANA

CONTENTS

CHAPTER 1. THE NATURE OF OPERATIONS RESEARCH'	1
1.1 The History of Operations Research	1
1.2 The Meaning of Operations Research	3
1.3 Models in Operations Research	4
1.4 Principles of Modeling	7
CHAPTER 2. LINEAR PROGRAMMING	13
2.1 Introduction	13
2.2 Formulation of Linear Programming Models	14
2.3 Graphical Solution of Linear Programs in Two Variables	23
2.4 Linear Program in Standard Form	27
2.5 Solving Systems of Linear Equations	31
2.6 Principles of the Simplex Method	33
2.7 Simplex Method in Tableau Form	36
2.8 Computational Problems	43
2.9 Finding a Feasible Basis	46
2.10 Computer Solution of Linear Programs	53
2.11 Additional Topics in Linear Programming	55
Recommended Readings	56
References	56
Exercises	57
CHAPTER 3. NETWORK ANALYSIS	65
3.1 Some Examples of Network Flow Problems	65
3.2 Transportation Problems	65
3.3 Assignment Problems	78
3.4 The Least-Time Transportation Problem'	84
3.5 Maximal-Flow Problems	87
3.6 Shortest-Route Problems	95
3.7 Project Management	101

Recommended Readings	118
^References	118
Exercises	119
CHAPTER 4. ADVANCED TOPICS IN LINEAR PROGRAMMING	125
4.1 The Revised Simplex Method	125
4.2 Duality Theory and its Applications	139
4.3 •The Dual Simplex Method	154
4.4 Sensitivity Analysis in Linear Programming	160
4.5 Parametric Programming	170
4.6 Integer Programming	176
Recommended Readings	190
References	190
•Exercises	190
CHAPTER 5. PROBABILITY REVIEW	199
5.1 Introduction	199
I BASIC CONCEPTS	199
5.2 Experiments, Sample Spaces, and Events	199
5.3 Probability	200
5.4' Random Variables	202
5.5 Probability Distributions	202
5.6 Joint, Marginal, and Conditional Distributions	205
5.7 Expectation	208
5.8 Variance and Other Moments	210
II DISCRETE PROBABILITY DISTRIBUTIONS	212
5.9 The Discrete Uniform Distribution	212
5.10 The Bernoulli Distribution	212
5.11 The Binomial Distribution	212
5.12 The Poisson Distribution	213
5.13 The Geometric Distribution	213
5.14 The Negative Binomial Distribution	214
III CONTINUOUS PROBABILITY DISTRIBUTIONS	215
5.15 The Continuous Uniform Distribution	215
5.16 The Normal Distribution	215
5.17 The Lognormal Distribution	217
5.18 The Negative Exponential Distribution	218
5.19 The Erlang Distribution	219
5.20 The Gamma Distribution	220
5.21 The Weibull Distribution	220
5.22 The Beta Distribution	221
IV USING DISTRIBUTIONS	222
5.23 Fitting Distributions to Data	222
Recommended Readings	224
References	225
Exercises	225

CHAPTER 6. RANDOM PROCESSES	229
6.1 Introduction	229
I DISCRETE TIME PROCESSES	230
6.2 An Example	230
6.3 Modeling the Process	230
6.4 Some Numerical Results	238
6.5 The Assumptions Reconsidered	239
6.6 Formal Definitions and Theory	240
6.7 Steady-State Probabilities	242
6.8 First-Passage and First-Return Probabilities	247
6.9 Classification Terminology	250
6.10 Transient Processes	256
II CONTINUOUS TIME PROCESSES	264
6.11 An Example	264
6.12 Formal Definitions and Theory	268
6.13 The Assumptions Reconsidered	272
6.14 Steady-State Probabilities	273
6.15 Birth-Death Processes	275
6.16 The Poisson Process	276
6.17 Conclusions	281
Recommended Readings	282
References	282
Exercises	282
CHAPTER 7. QUEUEING MODELS	289
7.1 Introduction	289
7.2 A Deterministic Model	289
7.3 Queue Parameters	291
7.4 The M/M/1 Queue	293
7.5 A Numerical Example	300
7.6 Limited Queue Capacity	301
7.7 Multiple Servers	304
7.8 Pooled versus Separate Servers	307
7.9 Finite Sources	308
7.10 Waiting Times	310
7.11 Queue Disciplines	312
7.12 Non-Markovian Queues	314
7.13 Conclusions	316
Recommended Readings	316
References	317
Exercises	317
CHAPTER 8. INVENTORY MODELS	321
8.1 Introduction	321
I DETERMINISTIC MODELS	321
8.2 The Classical Economic Order Quantity	322
8.3 A Numerical Example	325

8.4	Sensitivity Analysis	**	326
8.5	Non-zero Lead Time.		326
8.6	The EOQ with Shortages Allowed		327
8.7	The Production Lot-Size Model.	j.	328
8.8	The EOQ with Quantity Discounts.		329
8.9	The EOQ with Constraints.		330
8.10	Other Deterministic Inventory Models.		335
II PROBABILISTIC MODELS			336
8.11	The Newsboy, Problem: A Single Period Model.		336
8.12	A Lot Size, Reorder Point Model.		340
8.13	Some Numerical Examples.		347
8.14	Variable Lead Times.		351
8.15	The Importance of Selecting, the Right Model.		352
8.16	Conclusions.		354
	Recommended Readings.		354
	References.		354
	Exercises.		355
CHAPTER 9. SIMULATION			359
I BASIC CONCEPTS			359
9.1	Introduction.		359
9.2	The Philosophy, Development and Implementation of Simulation Modeling.		361
9.3	Design of Simulation Models.		366
II EXAMPLES OF SIMULATION MODELING			367
9.4	Sales of Life Insurance.		367
9.5	Production Line Maintenance.		369
III PSEUDO-RANDOM NUMBERS			377
9.6	Generation of Random Deviates.		377
9.7	The Uniform Distribution and its Importance to Simulation		377
9.8	Generation of Random Numbers.		378
	Properties of Uniformly Distributed Numbers.		379
	Midsquare Technique.		379
	Mid-Product Technique.		379
	Fibonacci Method.		379
9.9	The Logic in Generating Uniform Random Variates via a Congruential Method.		380
	Mixed Method.		381
	Multiplicative Method		382
	Quadratic Congruential Method.		382
9.10	Testing a Random Number Generator.		383
	The Frequency Test.		383
	The Gap Test.		385
	The Runs Test.		386
	The Poker Test.		387

IV TECHNIQUES FOR GENERATING RANDOM DEVIATES	388
9.11 The Inverse Transform Method	388
The Exponential Distribution	389
The Weibull Distribution	390
The Geometric Distribution	391
9.12 The Rejection Technique	391
The Beta Distribution	393
The Gamma Distribution	393
9.13 The Composition Method	396
The Poisson Distribution	397
The Erlang Distribution	397
The Binomial Distribution	398
9.14 Mathematical Derivation Technique	398
The Box and Muller Technique for Generating Normal Deviates	399
9.15 Approximation Techniques	400
9.16 Special Probability Distributions	401
The Chi-Square Distribution	401
The Student's <i>T</i> Distribution	402
The F Distribution	402
V SIMULATION LANGUAGES	403
9.17 An Overview	403
9.18 Comparison of Selected Existing Simulation Languages	405
GPSS III/GPSS (General Purpose Systems Simulator)	405
GASP II	406
SIMULA	407
DYNAMO	407
GASP IV	408
VI ADVANCED CONCEPTS IN SIMULATION ANALYSIS	409
9.19 Design of Simulation Experiments	409
9.20 Variance Reduction Techniques	410
9.21 Statistical Analysis of Simulation Output	411
9.22 Optimization of Simulation Parameters	411
9.23 Summary and Conclusions	412
Exercises	412
References	416
CHAPTER 10. DYNAMIC PROGRAMMING	419
I BASIC CONCEPTS	419
10.1 Introduction	419
10.2 Historical Background	419

II THE DEVELOPMENT OF DYNAMIC PROGRAMMING	420
10.3 Mathematical Description	420
10.4 Developing an Optimal Decision Policy	422
10.5 Dynamic Programming in Perspective.	424
III ILLUSTRATIVE EXAMPLES	425
10.6 A Problem in Oil Transport Technology	425
10.7 The Optimal Cutting Stock Problem	433
10.8 A Production Planning Problem	438
10.9 "A Problem in Inventory Control	442
10.10 Interchanging Optimization—Forward and Backward Recursion	446
IV CONTINUOUS STATE DYNAMIC PROGRAMMING	449
10.11 A Nonlinear Programming Problem	450
10.12 A Problem in Mutual Fund Investment Strategies	451
10.13 A Special Case of Linear Allocations in Continuous Dynamic Programming	454
V MULTIPLE STATE VARIABLES	456
10.14 The "Curse of Dimensionality"	456
10.15 A Nonlinear, Integer Programming Problem	456
10.16 Elimination of State Variables	460
10.17 Summary and Conclusions.	467
References	467
Exercises	468
CHAPTER 11. NONLINEAR PROGRAMMING	473
I BASIC CONCEPTS	473
11.1 Introduction	473
11.2 Taylors Series—Expansions; Necessary and Sufficiency Conditions	480
II UNCONSTRAINED OPTIMIZATION	489
11.3 Fibonacci and Golden Section Search	489
11.4 The Hooke and Jeeves Search Algorithm	497
11.5 Gradient Projection	501
III CONSTRAINED OPTIMIZATION PROBLEMS:	
EQUALITY CONSTRAINTS	507
11.6 Lagrange Multipliers	507
11.7 Equality Constrained Optimization: Constrained Derivatives	513
11.8 Projected Gradient Methods with Equality Constraints.	517
IV CONSTRAINED OPTIMIZATION PROBLEMS:	
INEQUALITY CONSTRAINTS	521
11.9 Non-Linear Optimization—The Kuhn-Tucker Conditions	521
11.10 Quadratic Programming	526

11.11 Complementary Pivot Algorithms	530
11.12 Separable Programming	539
V THE GENERAL NONLINEAR PROGRAMMING PROBLEM 546	
11.13 Nonlinear Objective Function Subject to Linear or Nonlinear Constraints: A Cutting Plane Algorithm	546
11.14 Optimization by Geometric Programming.	552
References	561
Exercises	562
APPENDICES 567	
INDEX 579	