

# **Statistical Power Analysis for the Social and Behavioral Sciences**

Basic and Advanced Techniques

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