Structural Equations with Latent Variables

KENNETH A. BOLLEN

Department of Sociology
The University of North Carolina at Chapel Hill
Chapel Hill, North Carolina

WILEY

A Wiley-Interscience Publication

JOHN WILEY & SONS

New York • Chichester • Brisbane • Toronto • Singapore

Contents

1. Introduction **1**

Historical Background, **4**

2. Model Notation, Covariances, and Path Analysis **10**

Model Notation, 10
Latent Variable Model, 11
Measurement Model, 16
Covariance, 21
Covariance Algebra, 21
Sample Covariances, 23
Path Analysis, 32
Path Diagrams, 32
Decomposition of Covariances and Correlations, 34
Total, Direct, and Indirect Effects, 36
Summary, 39

3. Causality and Causal Models **40**

Nature of Causality, 40
Isolation, 45
Association, 57
Direction of Causation, 61
Limitations of "Causal" Modeling, 67
Model-Data versus Model-Reality Consistency, 67
Experimental and Nonexperimental Research, 72

Criticisms of Structural Equation Models, 78
Summary, *19*

4. Structural Equation Models with Observed Variables 80

Model Specification, 80
Implied Covariance Matrix, 85
Identification, 88
/-Rule, 93
Null B Rule, 94
Recursive Rule, 95
Rank and Order Conditions, 98
Summary of Identification Rules, 103
Estimation, 104
Maximum Likelihood (ML), 107
Unweighted Least Squares (ULS), 111
Generalized Least Squares (GLS), 113
Other Estimators, 115
Empirical Examples, 116
Further Topics, 123
Standardized and Unstandardized Coefficients, 123
Alternative Assumptions for x, 126
Interaction Terms, 128
Intercepts, 129
Analysis of Variance (ANOVA) and Related Techniques, 130
Summary, 131
Appendix 4A Derivation of F_{ML} (y and x Multinomial), 131
Appendix 4B Derivation of F_{ML} (S Wishart Distribution), 134
Appendix 4C Numerical Solutions to Minimize Fitting Functions, 136
Starting Values, 137
Steps in the Sequence, 138
Stopping Criteria, 143
Appendix 4D Illustrations of LISREL and EQS Programs, 144

LISREL Program, 144
EQS Program, 148

5. The Consequences of Measurement Error 151

Univariate Consequences, 151
Bivariate and Simple Regression Consequences, 154
Consequences in Multiple Regression, 159
Correlated Errors of Measurement, 167
Consequences in Multiequation Systems, 168
Union Sentiment Example, 168
Unknown Reliabilities, 170
Objective and Subjective Socioeconomic Status (SES), 172
Summary, 175
Appendix 5A Illustrations of LISREL and EQS Programs, 176
LISREL Program, 176
EQS Program, 178

6. Measurement Models: The Relation between Latent and Observed Variables 179

Measurement Models, 179
Validity, 184
Content Validity, 185
Criterion Validity, 186
Construct Validity, 188
Convergent and Discriminant Validity, 190
Alternatives to Classical Validity Measures, 193
Summary, 206
Reliability, 206
Empirical Means of Assessing Reliability, 209
Alternatives to Classical Reliability Measures, 218
Summary, 221
Cause Indicators, 222
Summary, 223
Appendix 6A LISREL Program for the Multitrait-Multimethod Example, 223

7. Confirmatory Factor Analysis 226

Exploratory and Confirmatory Factor Analysis, 226
Model Specification, 233
Implied Covariance Matrix, 236
Identification, 238
/-Rule, 242
Three-Indicator Rules, 244
Two-Indicator Rules, 244
Summary of Rules, 246
An Insufficient Condition of Identification, 246
Empirical Tests of Identification, 246
Recommendations for Checking Identification, 251
Political Democracy Example, 251
Estimation, 254
Nonconvergence, 254
Model Evaluation, 256
Overall Model Fit Measures, 256
Component Fit Measures, 281
Does the Sympathy and Anger Model Fit?, 289
Comparisons of Models, 289
Likelihood Ratio (LR) Test, 292
Lagrangian Multiplier (LM) Test, 293
Wald (W) Test, 293
Summary of LR, LM, and W Tests, 295
Respecification of Model, 296
Theory and Substantive-Based Revisions, 296
Exploratory LR, LM, and W Tests, 298
Other Empirical Procedures, 303
Summary of Respecification Procedures, 304
Extensions, 305
Factor Score Estimation, 305
Latent Variable Means and Equation Intercepts, 306
Cause Indicators, 311
Higher-Order Factor Analyses, 313
Summary, 315
Appendix 7A Examples of Program Listings, 316

Example 1: Estimating Means and Intercepts, 316
Example 2: Estimating a Higher-Order Factor Analysis, 317

8. The General Model, Part I: Latent Variable and Measurement Models Combined 319

Model Specification, 319
Implied Covariance Matrix, 323
Identification, 326
/-Rule, 328
Two-Step Rule, 328
MIMIC Rule, 331
Summary of Identification Rules, 331
Industrialization and Political Democracy Example,- 332
Estimation and Model Evaluation, 333
Power of Significance Test, 338
Standardized and Unstandardized Coefficients, 349
Means and Equation Constants, 350
Comparing Groups, 355
Invariance of Intercepts and Means, 365
Missing Values, 369
Alternative Estimators of Covariance Matrix, 370
Explicit Estimation, 373
Systematic Missing Values, 375
Total, Direct, and Indirect Effects, 376
Specific Effects, 383
Summary, 389
Appendix 8A Asymptotic Variances of Effects, 390
Appendix 8B Listing of the LISREL VI Program for Missing Value Example, 393

9. The General Model, Part II: Extensions 395

/ Alternative Notations/Representations, 395
Equality and Inequality Constraints, 401
Quadratic and Interaction Terms, 403
Instrumental-Variable (IV) Estimators, 409
Distributional Assumptions, 415

Consequences, 416
Detection, 418
Correction, 425
Categorical Observed Variables, 433
Assumptions Violated and Consequences, 433
Robustness Studies, 434
Summary of Robustness to Categorization, 438
Corrective Procedures, 439
Summary, 446
Appendix 9A LISREL Program for Model in Figure 9.1(c), 447

Appendix A. Matrix Algebra Review **449**

Scalars, Vectors, and Matrices, 449
Matrix Operations, 450

Appendix B. Asymptotic Distribution Theory **466**

Convergence in Probability, 466
Convergence in Distributions, 468

References **471**

Index **489**