

---

# Stochastic Optimization Models in Finance

*Edited by*

**W. T. ZIEMBA**

*Faculty of Commerce and Business Administration  
The University of British Columbia  
Vancouver, Canada*

**R. G. VICKSON**

*Department of Management Sciences  
University of Waterloo  
Waterloo, Ontario, Canada*



ACADEMIC PRESS    New York    San Francisco    London  
*A Subsidiary of Harcourt Brace Jovanovich, Publishers*

# CONTENTS

<i>Preface</i>	<i>xiii</i>
<i>Acknowledgments</i>	<i>xvi</i>
PART I. MATHEMATICAL TOOLS	
<b>Introduction</b>	<b>3</b>
<b>1. Expected Utility Theory</b>	<b>11</b>
A General Theory of Subjective Probabilities and Expected Utilities	
Peter C. Fishburn	
<i>The Annals of Mathematical Statistics</i> <b>40</b> , 1419–1429 (1969)	<i>11</i>
<b>2. Convexity and the Kuhn–Tucker Conditions</b>	<b>23</b>
Pseudo-Convex Functions	
O. L. Mangasarian	
<i>Journal of SIAM Control</i> <b>A3</b> , 281–290 (1965)	<i>23</i>
Convexity, Pseudo-Convexity and Quasi-Convexity of Composite Functions	
O. L. Mangasarian	
<i>Cahiers du Centre d'Études de Recherche Opérationnelle</i>	
<b>12</b> , 114–122 (1970)	<i>33</i>
<b>3. Dynamic Programming</b>	<b>43</b>
Introduction to Dynamic Programming	
W. T. Ziemba	<i>43</i>
<b>Computational and Review Exercises</b>	<b>57</b>
<b>Mind-Expanding Exercises</b>	<b>67</b>

## PART II. QUALITATIVE ECONOMIC RESULTS

<b>Introduction</b>	<b>81</b>
<b>1. Stochastic Dominance</b>	<b>89</b>
The Efficiency Analysis of Choices Involving Risk	
G. Hanoch and H. Levy	
<i>The Review of Economic Studies</i> <b>36</b> , 335–346 (1969)	89
A Unified Approach to Stochastic Dominance	
S. L. Brummelle and R. G. Vickson	101
<b>2. Measures of Risk Aversion</b>	<b>115</b>
Risk Aversion in the Small and in the Large	
John W. Pratt	
<i>Econometrica</i> <b>32</b> , 122–136 (1964)	115
<b>3. Separation Theorems</b>	<b>131</b>
The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets	
John Lintner	
<i>The Review of Economics and Statistics</i> <b>47</b> , 13–37 (1965)	131
Separation in Portfolio Analysis	
R. G. Vickson	157
<b>Computational and Review Exercises</b>	<b>171</b>
<b>Mind-Expanding Exercises</b>	<b>183</b>

## PART III. STATIC PORTFOLIO SELECTION MODELS

<b>Introduction</b>	<b>203</b>
<b>1. Mean-Variance and Safety First Approaches and Their Extensions</b>	<b>215</b>
The Fundamental Approximation Theorem of Portfolio Analysis in Terms of Means, Variances and Higher Moments	
Paul A. Samuelson	
<i>The Review of Economic Studies</i> <b>37</b> , 537–542 (1970)	215
The Asymptotic Validity of Quadratic Utility as the Trading Interval Approaches Zero	
James A. Ohlson	221

Safety-First and Expected Utility Maximization in Mean-Standard Deviation Portfolio Analysis	
David H. Pyle and Stephen J. Turnovsky	
<i>The Review of Economics and Statistics</i> <b>52</b> , 75–81 (1970)	235
Choosing Investment Portfolios When the Returns Have Stable Distributions	
W. T. Ziemba	
“Mathematical Programming in Theory and Practice,” P. L. Hammer and G. Zoutendijk, eds., pp. 443–482. North-Holland, Amsterdam (1974)	243
<b>2. Existence and Diversification of Optimal Portfolio Policies</b>	<b>267</b>
On the Existence of Optimal Policies under Uncertainty	
Hayne E. Leland	
<i>Journal of Economic Theory</i> <b>4</b> , 35–44 (1972)	267
General Proof That Diversification Pays	
Paul A. Samuelson	
<i>Journal of Financial and Quantitative Analysis</i> <b>2</b> , 1–13 (1967)	277
<b>3. Effects of Taxes on Risk Taking</b>	<b>291</b>
The Effects of Income, Wealth, and Capital Gains Taxation on Risk-Taking	
J. E. Stiglitz	
<i>Quarterly Journal of Economics</i> <b>83</b> , 263–283 (1967)	291
Some Effects of Taxes on Risk-Taking	
B. Näslund	
<i>The Review of Economic Studies</i> <b>35</b> , 289–306 (1968)	313
<b>Computational and Review Exercises</b>	<b>331</b>
<b>Mind-Expanding Exercises</b>	<b>343</b>
<b>PART IV. DYNAMIC MODELS REDUCIBLE TO STATIC MODELS</b>	
<b>Introduction</b>	<b>367</b>
<b>1. Models That Have a Single Decision Point</b>	<b>373</b>
Investment Analysis under Uncertainty	
Robert Wilson	
<i>Management Science</i> <b>15</b> , B-650–B-664 (1969)	373
<b>CONTENTS</b>	<b>ix</b>

<b>2. Risk Aversion over Time Implies Static Risk Aversion</b>	<b>389</b>
Multi-period Consumption–Investment Decisions	
Eugene F. Fama	
<i>The American Economic Review</i> <b>60</b> , 163–174 (1970)	389

<b>3. Myopic Portfolio Policies</b>	<b>401</b>
-------------------------------------	------------

    On Optimal Myopic Portfolio Policies, with and without Serial Correlation of Yields

Nils H. Hakansson	
<i>The Journal of Business of the University of Chicago</i> <b>44</b> , 324–334 (1971)	401

<b>Computational and Review Exercises</b>	<b>413</b>
---	------------

<b>Mind-Expanding Exercises</b>	<b>417</b>
---------------------------------	------------

PART V. DYNAMIC MODELS

<b>Introduction</b>	<b>429</b>
---------------------	------------

    Appendix A. An Intuitive Outline of Stochastic Differential Equations and Stochastic Optimal Control

R. G. Vickson	453
---------------	-----

<b>1. Two-Period Consumption Models and Portfolio Revision</b>	<b>459</b>
--	------------

    Consumption Decisions under Uncertainty

Jacques H. Drèze and Franco Modigliani	
<i>Journal of Economic Theory</i> <b>5</b> , 308–335 (1972)	459

    A Dynamic Model for Bond Portfolio Management

Stephen P. Bradley and Dwight B. Crane	
<i>Management Science</i> <b>19</b> , 139–151 (1972)	487

<b>2. Models of Optimal Capital Accumulation and Portfolio Selection</b>	<b>501</b>
--	------------

    Multi-period Consumption–Investment Decisions and Risk Preference

Edwin H. Neave	
<i>Journal of Economic Theory</i> <b>3</b> , 40–53 (1971)	501

    Lifetime Portfolio Selection by Dynamic Stochastic Programming

Paul A. Samuelson	
<i>The Review of Economics and Statistics</i> <b>51</b> , 239–246 (1969)	517

Optimal Investment and Consumption Strategies under Risk for a Class of Utility Functions	
Nils H. Hakansson <i>Econometrica</i> <b>38</b> , 587–607 (1970)	525
<b>3. Models of Option Strategy</b>	<b>547</b>
The Value of the Call Option on a Bond	
Gordon Pye <i>Journal of Political Economy</i> <b>74</b> , 200–205 (1966)	547
Evaluating a Call Option and Optimal Timing Strategy in the Stock Market	
Howard M. Taylor <i>Management Science</i> <b>14</b> , 111–120 (1967)	553
Bond Refunding with Stochastic Interest Rates	
Basil A. Kalymon <i>Management Science</i> <b>18</b> , 171–183 (1971)	563
Minimax Policies for Selling an Asset and Dollar Averaging	
Gordon Pye <i>Management Science</i> <b>17</b> , 379–393 (1971)	577
<b>4. The Capital Growth Criterion and Continuous-Time Models</b>	<b>593</b>
Investment Policies for Expanding Businesses Optimal in a Long-Run Sense	
Leo Breiman <i>Naval Research Logistics Quarterly</i> <b>7</b> , 647–651 (1960)	593
Portfolio Choice and the Kelly Criterion	
Edward O. Thorp <i>Business and Economics Statistics Section, Proceedings of the American Statistical Association</i> 215–224 (1971).	599
Optimum Consumption and Portfolio Rules in a Continuous-Time Model	
Robert C. Merton <i>Journal of Economic Theory</i> <b>3</b> , 373–413 (1971)	621
<b>Computational and Review Exercises</b>	<b>663</b>
<b>Mind-Expanding Exercises</b>	<b>677</b>
<i>Bibliography</i>	701
<i>Index</i>	715
CONTENTS	xi