

DYNAMICS OF ENDOGENOUS ECONOMIC GROWTH: A Case Study of the "Romer Model"

Gordon W. Schmidt
University of New South Wales
Sydney
Australia



ELSEVIER

2003

O

Amsterdam - Boston - Heidelberg - London - New York - Oxford - Paris - San Diego
San Francisco - Singapore - Sydney - Tokyo

Contents

Preface	xiii
Chapter 1 Introduction	1
1.1 Background	1
1.2 A short history of economic growth theory	2
1.2.1 Classical, Keynesian, and neoclassical growth theory	3
1.2.2 Early (neoclassical) endogenous growth theory	8
1.2.3 Modern endogenous growth theory	11
1.3 Structure and summary of the book	14
1.3.1 Chapter 2. Development of the dynamic system	15
1.3.1.1 Description of the model	15
1.3.1.2 The dynamic system	17
1.3.1.3 Calibration and sensitivity	18
1.3.2 Chapter 3: Dynamic behaviour of the system: approximations	19
1.3.2.1 Linearisation of the model	19
1.3.2.2 Phase-space analysis	20
1.3.2.3 A Solowian-Romer model	21
1.3.3 Chapter 4: Numerical integration	22
1.3.3.1 Application of numerical integration to the Romer system	23
1.3.3.2 Conversion to an initial-value problem	23
1.3.3.3 Finite differences and GEMPACK	24
1.3.3.4 Specification of the solution method	24
1.3.3.5 Numerical results of simulations	25
1.3.4 Chapter 5: Economic welfare and policy issues	26
1.3.4.1 Comparison of the social-optimum and market solutions	27
1.3.4.2 A subsidised-market solution	28
1.3.4.3 Adjustment of the subsidised-market model to the social-optimum steady state	29
1.3.5 Chapter 6: Concluding remarks	30
Chapter 2 The 'Romer Model': Development of the Dynamic System	31
2.1 Introduction	31
2.2 Description of the model	33

2.2.1	General	33
2.2.2	Research sector	35
2.2.3	Savings and capital accumulation	36
2.2.4	Final output sector	37
2.2.5	Capital goods producing sector	39
2.2.6	Price of technology	43
2.2.7	Allocation of human capital	44
2.2.8	Consumption	45
2.3	The dynamic system	47
2.3.1	Transversality conditions	47
2.3.2	Condensation of the equations	49
2.3.3	Asymptotic dynamics and the steady-state equilibrium	51
2.4	Calibration and sensitivity	56
2.4.1	Calibration of the model	56
2.4.1.1	Parameter constraints	58
2.4.1.2	Endogenous variable constraints	61
2.4.1.3	Determining the benchmark parameter set	62
2.4.2	Sensitivity of the steady state to parameter values	64
Appendix 2.1	Production technology and the nature of capital	69
A2.1.1	Production technology	69
A2.1.1.1	Specialised capital, increasing returns and technical change	69
A2.1.1.2	Marginal products and factor returns	73
A2.1.2	The nature of capital	76
Appendix 2.2	Dynamic maximisation problems	79
A2.2.1	General theory	79
A2.2.2	Maximising monopoly profits	84
A2.2.3	Maximising consumer welfare	88
Appendix 2.3	Transversality conditions	89
Appendix 2.4	A second steady state	92
Appendix 2.5	Derivation of some supplementary variables	94
Appendix 2.6	Preliminary transitional dynamics	97
A2.6.1	A 25% rise in the rate of depreciation of capital δ	98
A2.6.2	A 10% rise in the production cost of capital T_1 , and a 20% rise in the quantity of ordinary labour L	100
Chapter 3	Dynamic Behavior of the System: Linearisation, Phase Space Analysis, and Development of an Abridged Model	103
3.1	Introduction	103
3.2	The linearised Romer model	104

3.2.1	Linearisation of the model and its dynamic solution	104
3.2.2	A simulation of the transient dynamics of the linearised model	109
3.2.3	Speed of convergence	115
3.3	Phase-space analysis	119
3.3.1	Phase surfaces and phase-space regions	120
3.3.2	The saddle path in phase space	125
3.3.3	Schematic phase-space analysis	129
3.4	A 'Solowian-Romer' model	133
3.4.1	Development and specification	133
3.4.2	Numerical integration of the Solowian-Romer system	136
3.4.2.1	An unanticipated and sustained 10% rise in parameter γ	138
3.4.2.2	An anticipated and sustained 15% rise in parameter t_s	141
3.4.2.3	A temporary (five year) fall of 20% in parameter a	145
3.4.3	Phase-space of the Solowian-Romer system	148
3.4.4	Phase-space analysis of the Solowian-Romer dynamics	151
3.4.4.1	An unanticipated and sustained 10% rise in parameter γ	151
3.4.4.2	An anticipated and sustained 15% rise in parameter δ	152
3.4.4.3	A temporary (five year) fall of 20% in parameter a	154
Appendix 3.1	Linearisation of the Romer model	156
Appendix 3.2	Solution of a first-order system of linear differential equations	160
A3.2.1	General solution	160
A3.2.2	Necessary conditions for equilibrium	162
A3.2.3	The complete solution	166
A3.2.4	The oscillatory effect of complex-valued roots	166
Appendix 3.3	Eigenvalues and eigenvectors of a linear dynamic system	167
A3.3.1	Eigenvalues of a 3×3 matrix ft	167
A3.3.2	Eigenvectors of a 3×3 matrix ft	177
Appendix 3.4	Anticipated shocks: Analytic determination with the linearised model	178
A3.4.1	Algebraic derivation	178
A3.4.2	Computations of adjustment paths	181
A3.4.2.1	A sustained 15% rise in parameter t_s	181
A3.4.2.2	A temporary (five year) fall of 20% in parameter a	181

Appendix 3.5	Log-linearisation of the Romer model	187
A3.5.1	Deriving the log-linear coefficients matrix	187
A3.5.2	Speed of convergence in the log-linear model	189
Appendix 3.6	Phase-space surfaces of the Romer model	192
Appendix 3.7	Equivalence of the Solowian-Romer model and the full Romer model	204
Appendix 3.8	Log-linearisation of the Solowian-Romer model	212
Chapter 4	Numerical Integration	215
4.1	General application to the Romer system	215
4.2	Conversion to an initial value problem	220
4.2.1	Time elimination method	220
4.2.2	Eigenvector-backward integration method	223
4.3	Finite differences and the GEMPACK software	224
4.3.1	Method of finite differences	225
4.3.2	GEMPACK and its application to the Romer model	227
4.4	Specification of the solution method	230
4.4.1	Simulating unanticipated shocks	231
4.4.2	Simulating anticipated shocks	232
4.4.3	The solution method: finite differences and GEMPACK	233
4.5	Numerical results of simulations	234
4.5.1	An unanticipated rise in the profit share of income	235
4.5.2	An anticipated rise in the productivity of researchers	244
4.5.3	A temporary rise in the ordinary-labour share of wages	255
4.5.4	Comparison of the linearised, Solowian- and full non-linear Romer model results	261
4.5.5	An immigration program to raise the level of human capital	262
4.5.6	A sudden, temporary reduction of the capital stock	269
4.5.7	A gradual loss of human capital from research	274
Appendix 4.1	Numerical integration methods for initial-value problems	282
Appendix 4.2	Computing the saddle path of the Romer model as the solution to an 'initial value' problem	285
A4.2.1	Time-elimination method	285
A4.2.2	Eigenvector backward-integration method	302
Appendix 4.3	Specification of the finite differences-GEMPACK method of numerical integration	313
A4.3.1	General	313
A4.3.2	Empirical performances on the Romer model	314
A4.3.3	TABLO input and Command files for GEMPACK	321

Appendix 4.4	Comparison of the computed dynamic behaviours of the linearised Romer model, the Solowian-Romer model and the full non-linear Romer model	332
A4.4.1	An unanticipated and sustained 10% rise in parameter γ	347
A4.4.2	An anticipated and sustained 15% rise in parameter ℓ	348
A4.4.3	An unanticipated but temporary 20% fall in parameter a	348
Chapter 5	Economic Welfare and Policy Issues	351
5.1	Sub-optimality of the market solution	351
5.1.1	Social planning solution to the Romer model	354
5.1.2	Comparison of the social planning and market solutions	358
5.2	Policy implications	368
5.2.1	A subsidised-market solution	369
5.2.1.1	Production-side distortions	369
5.2.1.2	Externalities in research	371
5.2.1.3	Subsidised stationary dynamic system	372
5.2.1.4	Steady state of the subsidised-market solution	374
5.2.2	Optimum rates of subsidy	375
5.3	Adjustment to the social-optimum steady state	379
5.3.1	Unanticipated/single point of time imposition of the optimum subsidies	380
5.3.2	Unanticipated imposition of s_K ; and delayed, anticipated implementation of s_{AK}	386
5.3.3	Unanticipated imposition of s_{AK} ; and delayed, anticipated implementation of s_K	391
5.3.4	Dynamic effects of different methods of implementation	396
Appendix 5.1	A social planning solution to the Romer Model	397
A5.1.1	Derivation of the dynamic system and its steady state	397
A5.1.2	Calculation of the speed of convergence through linearisation	403
Appendix 5.2	TABLO input files for the 'social-optimum' and the 'subsidised-market' Romer systems	405
Appendix 5.3	Limiting levels and shares of human capital	416
Appendix 5.4	Interest rates, capital goods intensities, and technology prices in the market and optimal steady states	421
A5.4.1	Interest.rates	421
A5.4.2	Capital goods intensities and the price of technology	422

Appendix 5.5	Alternative subsidisation strategies to attain the social-optimum steady state.	429
A5.5.1	The production side distortions.	430
	A5.5.1.I Subsidising the rental costs of specialised capital.	430
	A5.5.1.2 Subsidising the manufacture of final output	431
A5.5.2	The research externality distortion	432
	A5.5.2.1 Subsidising the wages of researchers.	432
	A5.5.2.2 Subsidising the 'accumulation of research'	434
Appendix 5.6	Dynamic impact of the optimum subsidies.	435
A5.6.1	Savings and designs subsidies: (s_K, s_{AK}).	435
	A5.6.1.1 Impact of optimum subsidy s_K	439
	A5.6.1.2 Impact of optimum subsidy s_{AK}	441
A5.6.2	Monopoly correction and designs subsidies: Ox, *AX) or (s_y, s_{AY}) ...,'	442
Chapter 6	Concluding Remarks and Policy Implications	447
6.1	Conclusions regarding the dynamics.	447
	6.1.1 Technical methods.	447
	6.1.2 Dynamic results.	449
6.2	Policy implications	452
	6.2.1 Policy implications from the transitional dynamics.	453
	6.2.2 Policy implications from the sub-optimality of the model ...	455
6.3	Directions for further research	458
Bibliography.	463
Author index.	473
Subject index.	475