

# AN INTRODUCTION TO MULTILEVEL MODELING TECHNIQUES

MLM and SEM Approaches Using  
Mplus

Third Edition

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