

# Modeling Aggregate Behavior and Fluctuations in Economics

Stochastic Views of Interacting Agents

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