

Series/Number 07-150

DIFFERENTIAL EQUATIONS A Modeling Approach

Courtney Brown *Emory University*



CONTENTS

Series Editor's Introduction Acknowledgments		
	Theoretical Reasons for Using Differential Equations in the Social Sciences	3
	An Example	5
	The Use of Differential Equations in the	5
	Natural and Physical Sciences	7
	Deterministic Versus Probabilistic Differential	
	Equation Models	8
	What Is a Differential Equation?	11
	What This Book Is and Is Not	15
2.	First-Order Differential Equations	17
	Analytical Solutions to Linear First-Order	
	Differential Equations	18
	Solving First-Order Differential Equations	
	Using Separation of Variables	18
	Exponential Growth	20
	Exponential Decay	22
	Learning Curves and Noninteractive Diffusion	23
	Logistic Curve	25
	An Example From Sociology	29
	Numerical Methods Used to Solve Differential Equations	30
	Eider's Method .	31
,	Heun 's Method	34
	The Fourth-Order Runge-Kutta Method	35
	Summary	36
	Chapter 2 Appendix	37

3.	Systems of First-Order Differential Equations	40
	The Predator-Prey Model	41
	The Phase Diagram	44
	Equilibria Within Phase Diagrams	46
	Vector Field and Direction Field Diagrams	48
	The Equilibrium Marsh and Flow Diagrams	53
	Summary	55
	Chapter 3 Appendix	56
4.	Some Classic Social Science Examples	
	of First-Order Systems	59
	Richardson's Arms Race Model	60
	Lanchester's Combat Models	65
	Scenario One	66
	Scenario Two	67
	Scenario Three	67
	Rapoport's Production and Exchange Model	67
	Summary	69
5.	Transforming Second-Order and Nonautonomous	
	Differential Equations Into Systems of First-Order	
	Differential Equations •	70
	Second- and Higher-Order Differential Equations	72
	An Example	73
	Nonautonomous Differential Equations	74
	Summary	75
6.	Stability Analyses of Linear Differential	
	Equation Systems	75
	A Motivating Example of How Stability Can	
	Dramatically Change in One System	76
	Scalar Methods	77
	Matrix Methods	81
	Equilibrium Categories	85
	Unstable Nodes	85
	Stable Nodes	86
	Saddle Points	87
	Unstable Spirals	88
	Stable Spirals	89
	Ellipses	90
	Summarizing the Stability Criteria	91

7.	Stability Analyses of Nonlinear	
	Differential Equation Systems	93
	The Jacobian	93
	An Example	96
	Summary	97
8.	Frontiers of Exploration	97
References		100
Index		102
About the Author		106