

Econometric Analysis

Contents

Preface

List of Examples

List of Theorems and Useful Results

List of Definitions

CHAPTER 1 Introduction 1

- 1.1. Econometrics 1
- 1.2. Econometric Modeling 2
- 1.3. Theoretical and Applied Econometrics 3
- 1.4. Plan of the Book 4

CHAPTER 2 Matrix Algebra 6

- 2.1. Introduction 6
- 2.2. Some Terminology 6
- 2.3. Algebraic Manipulation of Matrices 8
 - 2.3.1. Equality of Matrices 8
 - 2.3.2. Transposition 8
 - 2.3.3. Matrix Addition 9
 - 2.3.4. Matrix Multiplication 9
 - 2.3.5. Sums of Values 12
 - 2.3.6. A Useful Idempotent Matrix 13
- 2.4. Geometry of Matrices 15
 - 2.4.1. Vector Spaces 15
 - 2.4.2. Linear Combinations of Vectors and Basic Vectors 17
 - 2.4.3. Linear Dependence 19
 - 2.4.4. Subspaces 19
 - 2.4.5. Rank of a Matrix 20
 - 2.4.6. Determinant of a Matrix 23
 - 2.4.7. A Least Squares Problem 26



2.5.	Solution of a System of Linear Equations	28
2.5.1.	<i>Systems of Linear Equations</i>	28
2.5.2.	<i>Inverse Matrices</i>	29
2.5.3.	<i>Nonhomogeneous Systems of Equations</i>	31
2.6.	Partitioned Matrices	32
2.6.1.	<i>Addition and Multiplication of Partitioned Matrices</i>	32
2.6.2.	<i>Determinants of Partitioned Matrices</i>	33
2.6.3.	<i>Inverses of Partitioned Matrices</i>	33
2.6.4.	<i>Deviations from Means</i>	33
2.6.5.	<i>Kronecker Products</i>	34
2.7.	Characteristic Roots and Vectors	35
2.7.1.	<i>The Characteristic Equation</i>	35
2.7.2.	<i>Characteristic Vectors</i>	36
2.7.3.	<i>General Results for Characteristic Roots and Vectors</i>	37
2.7.4.	<i>Diagonalization and Spectral Decomposition of a Matrix</i>	38
2.7.5.	<i>Rank of a Matrix</i>	38
2.7.6.	<i>Condition Number of a Matrix</i>	39
2.7.7.	<i>Trace of a Matrix</i>	40
2.7.8.	<i>Determinant of a Matrix</i>	41
2.7.9.	<i>Powers of a Matrix</i>	41
2.7.10.	<i>Idempotent Matrices</i>	43
2.7.11.	<i>Factoring a Matrix</i>	43
2.7.12.	<i>The Generalized Inverse of a Matrix</i>	44
2.8.	Quadratic Forms and Definite Matrices	46
2.8.1.	<i>Nonnegative Definite Matrices</i>	47
2.8.2.	<i>Idempotent Quadratic Forms</i>	47
2.8.3.	<i>Ranking Matrices</i>	48
2.9.	Calculus and Matrix Algebra	49
2.9.1.	<i>Differentiation and the Taylor Series</i>	49
2.9.2.	<i>Optimization</i>	53
2.9.3.	<i>Constrained Optimization</i>	55
2.9.4.	<i>Transformations</i>	57
	Exercises	59

CHAPTER 3 Probability and Distribution Theory 62

3.1.	Introduction	62
3.2.	Random Variables	62
3.2.1.	<i>Probability Distributions</i>	63
3.2.2.	<i>Cumulative Distribution Function</i>	63
3.3.	Expectations of a Random Variable	64
3.4.	Some Specific Probability Distributions	67
3.4.1.	<i>The Normal Distribution</i>	67
3.4.2.	<i>The Chi-Squared, t, and F Distributions</i>	68
3.4.3.	<i>Distributions with Large Degrees of Freedom</i>	70
3.4.4.	<i>Size Distributions—The Lognormal Distribution</i>	71
3.4.5.	<i>The Gamma and Exponential Distributions</i>	71



3.4.6.	<i>The Beta Distribution</i>	71
3.4.7.	<i>The Logistic Distribution</i>	72
3.4.8.	<i>Discrete Random Variables</i>	72
3.5.	The Distribution of a Function of a Random Variable	73
3.6.	Joint Distributions	75
3.6.1.	<i>Marginal Distributions</i>	76
3.6.2.	<i>Expectations in a Joint Distribution</i>	76
3.6.3.	<i>Covariance and Correlation</i>	77
3.6.4.	<i>Distribution of a Function of Bivariate Random Variables</i>	78
3.7.	Conditioning in a Bivariate Distribution	79
3.7.1.	<i>Regression—The Conditional Mean</i>	80
3.7.2.	<i>Conditional Variance</i>	81
3.7.3.	<i>Relationships Among Marginal and Conditional Moments</i>	81
3.7.4.	<i>The Analysis of Variance</i>	84
3.8.	The Bivariate Normal Distribution	85
3.9.	Multivariate Distributions	86
3.9.1.	<i>Moments</i>	86
3.9.2.	<i>Sets of Linear Functions</i>	87
3.9.3.	<i>Nonlinear Functions</i>	88
3.10.	The Multivariate Normal Distribution	89
3.10.1.	<i>Marginal and Conditional Normal Distributions</i>	89
3.10.2.	<i>Linear Functions of a Normal Vector</i>	91
3.10.3.	<i>Quadratic Forms in a Standard Normal Vector</i>	91
3.10.4.	<i>The F Distribution</i>	93
3.10.5.	<i>A Full Rank Quadratic Form</i>	93
3.10.6.	<i>Independence of a Linear and a Quadratic Form</i>	94
	Exercises	95

CHAPTER 4 Statistical Inference 101

4.1.	Introduction	101
4.2.	Samples and Sampling Distributions	101
4.2.1.	<i>Random Sampling</i>	101
4.2.2.	<i>Descriptive Statistics</i>	102
4.2.3.	<i>Sampling Distributions</i>	105
4.3.	Point Estimation of Parameters	106
4.3.1.	<i>Estimation in a Finite Sample</i>	108
4.3.2.	<i>Efficient Unbiased Estimation</i>	111
4.4.	Large-Sample Distribution Theory	115
4.4.1.	<i>Convergence in Probability</i>	115
4.4.2.	<i>Convergence in Distribution—Limiting Distributions</i>	119
4.4.3.	<i>Asymptotic Distributions</i>	125
4.4.4.	<i>Asymptotic Distribution of a Nonlinear Function</i>	127
4.4.5.	<i>Asymptotic Expectations</i>	127
4.5.	Efficient Estimation—Maximum Likelihood	129
4.5.1.	<i>Properties of Maximum Likelihood Estimators</i>	133



4.5.2.	<i>Estimating the Asymptotic Variance of the Maximum Likelihood Estimator</i>	138
4.6.	Two-Step Estimation	140
4.7.	Consistent Estimation—The Method of Moments	145
4.7.1.	<i>Random Sampling and Estimating the Parameters of Distributions</i>	145
4.7.2.	<i>Computing the Variance of a Method of Moments Estimator</i>	149
4.8.	Interval Estimation	153
4.9.	Hypothesis Testing	155
4.9.1.	<i>Testing Procedures</i>	156
4.9.2.	<i>Tests Based on Confidence Intervals</i>	158
4.9.3.	<i>Three Asymptotically Equivalent Test Procedures</i>	159
4.9.3a.	<i>The Likelihood Ratio Test</i>	161
4.9.3b.	<i>The Wald Test</i>	162
4.9.3c.	<i>The Lagrange Multiplier Test</i>	165
4.9.4.	<i>An Example of the Test Procedures</i>	167
	Exercises	169

CHAPTER 5 Computation and Optimization 173

5.1.	Introduction	173
5.2.	Digital Computing	174
5.3.	Data Input and Generation	175
5.3.1.	<i>Data Input</i>	176
5.3.2.	<i>Generating Pseudo-Random Numbers</i>	176
5.3.2a.	<i>Sampling from a Standard Uniform $U[0, 1]$ Population</i>	177
5.3.2b.	<i>Sampling from Continuous Distributions</i>	178
5.3.2c.	<i>Sampling from a Multivariate Normal Population</i>	179
5.3.2d.	<i>Sampling from a Discrete Population</i>	179
5.3.2e.	<i>The Gibbs Sampler</i>	179
5.3.3.	<i>Monte Carlo Studies</i>	182
5.3.4.	<i>Bootstrapping</i>	184
5.4.	Computation in Econometrics	185
5.4.1.	<i>Summation and Multiplication</i>	185
5.4.2.	<i>Computing Integrals</i>	187
5.4.2a.	<i>The Standard Normal Cumulative Distribution Function</i>	187
5.4.2b.	<i>The Gamma and Related Functions</i>	189
5.4.2c.	<i>Approximating Integrals by Quadrature</i>	190
5.4.2d.	<i>Monte Carlo Integration</i>	192
5.4.2e.	<i>Multivariate Normal Probabilities and Simulated Moments</i>	195
5.4.3.	<i>Computing Derivatives</i>	197
5.5.	Optimization	198
5.5.1.	<i>Algorithms</i>	200
5.5.2.	<i>Gradient Methods</i>	202
5.5.3.	<i>Aspects of Maximum Likelihood Estimation</i>	205
5.5.4.	<i>Optimization with Constraints</i>	210

5.5.5. Some Practical Considerations 212

5.6. Examples 214

Exercises 218

CHAPTER 6 The Classical Multiple Linear Regression Model – Specification and Estimation 220

6.1. Introduction 220

6.2. The Linear Model 220

6.3. Assumptions of the Classical Linear Regression Model 225

6.3.1. *Linearity of the Regression Model* 2256.3.2. *Full Rank* 2306.3.3. *Regression* 2316.3.4. *Spherical Disturbances* 2326.3.5. *Nonstochastic Regressors* 2336.3.6. *Normality* 2346.3.7. *Summary* 235

6.4. Least Squares Regression 236

6.4.1. *The Least Squares Coefficient Vector* 2366.4.2. *Algebraic Aspects of the Least Squares Solution* 2436.4.3. *Partitioned Regression and Partial Regression* 2456.4.4. *Partial Regression and Partial Correlation Coefficients* 248

6.5. Goodness of Fit and the Analysis of Variance 250

6.6. Statistical Properties of the Least Squares Estimator in Finite Samples 257

6.6.1. *Nonstochastic Regressors* 2586.6.2. *Stochastic Regressors* 2606.6.3. *Normality and the Distribution of $\hat{\beta}$* 2626.6.4. *Estimating σ^2 and the Variance of $\hat{\beta}$* 2636.6.5. *Testing a Hypothesis About a Coefficient* 2646.6.6. *Confidence Intervals for Parameters* 2666.6.7. *Testing the Significance of the Regression* 2686.6.8. *Test Statistics with Stochastic X and Normal ϵ* 270

6.7. Large-Sample Results for the Classical Regression Model 270

6.7.1. *The Finite-Sample Properties of Least Squares* 2716.7.2. *Consistency of the Least Squares Estimator of β* 2726.7.3. *Asymptotic Normality of the Least Squares Estimator* 2756.7.4. *Consistency of s^2 and the Estimator of $\text{Asy. Var}[\hat{\beta}]$* 2776.7.5. *Asymptotic Distribution of a Function of $\hat{\beta}$ —The Delta Method* 2786.7.6. *Asymptotic Behavior of the Standard Test Statistics* 2806.7.7. *Independent Observations on Stochastic Regressors* 2846.7.8. *Correlation Between x_i and ϵ_i —Instrumental Variables Estimation* 2886.7.9. *Heterogeneity in the Distributions of x_i* 2956.7.10. *Dependent Observations* 298

6.8.	Normally Distributed Disturbances	299
6.8.1.	<i>Asymptotic Efficiency—Maximum Likelihood Estimation</i>	300
6.8.2.	<i>Stochastic Regressors</i>	302
6.8.3.	<i>Wald, Lagrange Multiplier, and Likelihood Ratio Test Statistics</i>	303
6.8.4.	<i>Cases in Which Least Squares Is Inefficient</i>	306
6.8.5.	<i>Alternative Estimation Criteria</i>	307
6.8.6.	<i>Detecting Departures from Normality</i>	308
6.9.	Bayesian Estimation	311
6.9.1.	<i>Bayesian Analysis of the Classical Regression Model</i>	312
6.9.2.	<i>Point Estimation</i>	316
6.9.3.	<i>Interval Estimation</i>	317
6.9.4.	<i>Estimation with an Informative Prior Density</i>	317
6.9.5.	<i>Hypothesis Testing</i>	320
	Exercises	323

CHAPTER 7 Inference and Prediction 333

7.1.	Introduction	333
7.2.	Testing Restrictions	333
7.2.1.	<i>Two Approaches to Testing Hypotheses</i>	334
7.2.2.	<i>Testing a Set of Linear Restrictions</i>	334
7.2.3.	<i>Testing One Linear Restriction</i>	335
7.2.4.	<i>Testing J Linear Restrictions</i>	337
7.2.5.	<i>A Test Based on a Confidence Region</i>	339
7.2.6.	<i>Nonnormal Disturbances</i>	341
7.3.	The Restricted Least Squares Estimator	341
7.4.	A Test Based on Loss of Fit	343
7.5.	Examples and Some General Procedures	344
7.6.	Tests of Structural Change	349
7.6.1.	<i>Different Parameter Vectors</i>	350
7.6.2.	<i>Different Constant Terms</i>	351
7.6.3.	<i>Change in a Subset of Coefficients</i>	352
7.6.4.	<i>Insufficient Observations</i>	353
7.7.	Tests of Structural Change with Unequal Variances	354
7.8.	Alternative Tests of Model Stability	355
7.9.	Testing Nonlinear Restrictions	360
7.10.	Choosing Between Nonnested Models	364
7.10.1.	<i>An Encompassing Model</i>	365
7.10.2.	<i>The J Test</i>	365
7.10.3.	<i>The Cox Test</i>	366
7.11.	Prediction	369
7.11.1.	<i>A Convenient Method of Computing the Forecasts</i>	370
7.11.2.	<i>Measuring the Accuracy of Forecasts</i>	372
	Exercises	375

**CHAPTER 8 Functional Form, Nonlinearity, and Specification 379**

- 8.1. Introduction 379
- 8.2. Dummy Variables 379
 - 8.2.1. Comparing Two Means 379
 - 8.2.2. Binary Variables in Regression 382
 - 8.2.3. Several Categories 383
 - 8.2.4. Several Groupings 384
 - 8.2.5. Threshold Effects 385
 - 8.2.6. Interactions and Spline Regression 387
- 8.3. Nonlinearity in the Variables 390
 - 8.3.1. Functional Forms 390
 - 8.3.2. Identifying Nonlinearity 393
 - 8.3.3. Intrinsic Linearity and Identification 395
- 8.4. Specification Analysis 399
 - 8.4.1. Selection of Variables 399
 - 8.4.2. Omission of Relevant Variables 401
 - 8.4.3. Inclusion of Irrelevant Variables 404
- 8.5. Biased Estimators and Pretest Estimators 405
 - 8.5.1. The Mean-Squared-Error Test 405
 - 8.5.2. Pretest Estimators 408
 - 8.5.3. Inequality Restrictions 411

Exercises 413

CHAPTER 9 Data Problems 418

- 9.1. Introduction 418
- 9.2. Multicollinearity 418
 - 9.2.1. Perfect Collinearity 419
 - 9.2.2. Near Multicollinearity 419
 - 9.2.3. The Symptoms of Multicollinearity 420
 - 9.2.4. Suggested Remedies for the Multicollinearity Problem 423
 - 9.2.5. Conclusion 427
- 9.3. Missing Observations 427
- 9.4. Grouped Data 432
- 9.5. Measurement Error and Proxy Variables 435
 - 9.5.1. One Badly Measured Variable 436
 - 9.5.2. Multiple Regression with Measurement Error 439
 - 9.5.3. The Method of Instrumental Variables 440
 - 9.5.4. Proxy Variables 442
 - 9.5.5. A Specification Test for Measurement Error 443
- 9.6. Regression Diagnostics and Influential Data Points 444

Exercises 446

CHAPTER 10 Nonlinear Regression Models 450

- 10.1. Introduction 450
- 10.2. Nonlinear Regression Models 450

10.2.1.	<i>The Linearized Regression</i>	452
10.2.2.	<i>The Nonlinear Least Squares Estimator</i>	453
10.2.3.	<i>Computing the Nonlinear Least Squares Estimator</i>	455
10.2.4.	<i>A Specification Test for Nonlinear Regressions: Testing for Linear Versus Log-Linear Specification</i>	459
10.2.5.	<i>Nonlinear Instrumental Variables Estimation</i>	462
10.2.6.	<i>Two-Step Nonlinear Least Squares Estimation</i>	465
10.3.	Parametric Transformations of the Dependent Variable	473
10.4.	The Box–Cox Transformation	479
10.4.1.	<i>Transforming the Independent Variables</i>	480
10.4.2.	<i>Transforming the Model</i>	483
10.4.3.	<i>A Test for (Log-) Linearity</i>	486
10.5.	Hypothesis Testing and Parametric Restrictions	487
10.5.1.	<i>An Asymptotically Valid F Test</i>	488
10.5.2.	<i>Wald Test</i>	488
10.5.3.	<i>Likelihood Ratio Test</i>	489
10.5.4.	<i>Lagrange Multiplier Test</i>	489
	Exercises	494

CHAPTER 11 Nonspherical Disturbances, Generalized Regression, and GMM Estimation 496

11.1.	Introduction	496
11.2.	Consequences for Least Squares Estimation	497
11.2.1.	<i>Finite-Sample Properties of Ordinary Least Squares</i>	498
11.2.2.	<i>Asymptotic Properties of Least Squares</i>	499
11.2.3.	<i>Asymptotic Properties of Nonlinear Least Squares</i>	501
11.2.4.	<i>Asymptotic Properties of the Instrumental Variables Estimator</i>	502
11.2.5.	<i>Robust Estimation of Asymptotic Covariance Matrices</i>	503
11.3.	Efficient Estimation	507
11.3.1.	<i>Generalized Least Squares (GLS)</i>	507
11.3.2.	<i>Maximum Likelihood Estimation</i>	509
11.4.	Estimation When Ω Is Unknown	511
11.4.1.	<i>Feasible Generalized Least Squares</i>	511
11.4.2.	<i>Maximum Likelihood Estimation</i>	513
11.5.	The Generalized Method of Moments (GMM) Estimator	517
11.5.1.	<i>Methods of Moments Estimators</i>	517
11.5.2.	<i>Generalizing the Method of Moments</i>	519
11.5.3.	<i>Computation of q and Using Nonoptimal Weighting Matrices</i>	524
11.5.4.	<i>Testing the Validity of the Moment Restrictions</i>	526
11.5.5.	<i>GMM Estimation of Econometric Models</i>	527
11.6.	Testing Hypotheses in the GMM Framework	532
11.6.1.	<i>GMM Counterparts to the Wald, LM, and LR Tests</i>	532
11.6.2.	<i>Conditional Moment Tests</i>	534
	Exercises	536

CHAPTER 12 Heteroscedasticity 540

- 12.1. Introduction 540
- 12.2. Ordinary Least Squares Estimation 543
 - 12.2.1. *Inefficiency of Least Squares* 543
 - 12.2.2. *The Estimated Covariance Matrix of \hat{b}* 545
 - 12.2.3. *Estimating the Appropriate Covariance Matrix for Ordinary Least Squares* 547
- 12.3. Testing for Heteroscedasticity 549
 - 12.3.1. *White's General Test* 550
 - 12.3.2. *The Goldfeld–Quandt Test* 551
 - 12.3.3. *The Breusch–Pagan/Godfrey Test* 552
 - 12.3.4. *Testing for Groupwise Heteroscedasticity* 553
 - 12.3.5. *Tests Based on Regressions—Glesjer's (1969) Test* 554
- 12.4. Generalized Least Squares When Ω Is Known 555
- 12.5. Estimation When Ω Contains Unknown Parameters 558
 - 12.5.1. *Two-Step Estimation* 558
 - 12.5.2. *Maximum Likelihood Estimation* 562
- 12.6. General Conclusions 567
- 12.7. Autoregressive Conditional Heteroscedasticity 569
- Exercises 573

CHAPTER 13 Autocorrelated Disturbances 577

- 13.1. Introduction 577
- 13.2. The Analysis of Time-Series Data 580
- 13.3. Disturbance Processes 582
 - 13.3.1. *Characteristics of Disturbance Processes* 582
 - 13.3.2. *AR(1) Disturbances* 584
- 13.4. Least Squares Estimation 586
 - 13.4.1. *OLS Estimation with Lagged Dependent Variables* 586
 - 13.4.2. *Efficiency of Least Squares* 587
 - 13.4.3. *Estimating the Variance of the Least Squares Estimator* 590
- 13.5. Testing for Autocorrelation 591
 - 13.5.1. *The Durbin–Watson Test* 591
 - 13.5.2. *Other Testing Procedures* 594
 - 13.5.3. *Testing in the Presence of Lagged Dependent Variables* 596
- 13.6. Efficient Estimation When Ω Is Known 597
 - 13.6.1. *Generalized Least Squares* 597
 - 13.6.2. *Maximum Likelihood Estimation* 599
- 13.7. Estimation When Ω Is Unknown 600
 - 13.7.1. *AR(1) Disturbances* 600
 - 13.7.2. *AR(2) Disturbances* 605
 - 13.7.3. *Estimation with a Lagged Dependent Variable* 605
- 13.8. Forecasting in the Presence of Autocorrelation 607
- Exercises 609



CHAPTER 14 Models for Panel Data	612
14.1. Introduction	612
14.2. Panel Data Models	612
14.3. Fixed Effects	615
14.3.1. <i>Testing the Significance of the Group Effects</i>	617
14.3.2. <i>The Within and Between Groups Estimators</i>	618
14.3.3. <i>Fixed Time and Group Effects</i>	621
14.3.4. <i>Unbalanced Panels and Fixed Effects</i>	622
14.4. Random Effects	623
14.4.1. <i>Generalized Least Squares</i>	624
14.4.2. <i>Feasible Generalized Least Squares When Ω Is Unknown</i>	626
14.4.3. <i>Testing for Random Effects</i>	628
14.4.4. <i>Hausman's Test for Fixed or Random Effects</i>	632
14.4.5. <i>Unbalanced Panels and Random Effects</i>	634
14.5. Heteroscedasticity and Robust Covariance Estimation	635
14.5.1. <i>Robust Estimation of the Fixed Effects Model</i>	635
14.5.2. <i>Heteroscedasticity in the Random Effects Model</i>	636
14.6. Autocorrelation	638
14.7. Dynamic Models	640
14.8. Conclusions	641
Exercises	642

CHAPTER 15 Systems of Regression Equations	648
15.1. Introduction	648
15.2. Covariance Structures for Time-Series Cross-Section Data	651
15.2.1. <i>Cross-Sectional Heteroscedasticity</i>	653
15.2.2. <i>Cross-Sectional Correlation</i>	658
15.2.3. <i>Autocorrelation</i>	662
15.2.4. <i>Summary</i>	666
15.3. A Random Coefficients Model	669
15.4. The Seemingly Unrelated Regressions Model	674
15.4.1. <i>Generalized Least Squares</i>	675
15.4.2. <i>Feasible Generalized Least Squares</i>	676
15.4.3. <i>Maximum Likelihood Estimation</i>	681
15.4.3a. <i>Iterated FGLS</i>	681
15.4.3b. <i>Direct Maximum Likelihood Estimation</i>	682
15.4.4. <i>Autocorrelation</i>	687
15.5. Systems of Demand Equations—Singular Systems	688
15.6. Flexible Functional Forms—The Translog Cost Function	693
15.7. Nonlinear Systems and GMM Estimation	698
15.7.1. <i>GLS Estimation</i>	699
15.7.2. <i>Maximum Likelihood Estimation</i>	701
15.7.3. <i>GMM Estimation</i>	702
Exercises	703

**CHAPTER 16 Simultaneous Equations Models 708**

- 16.1. Introduction 708
- 16.2. Fundamental Issues in Simultaneous Equations Models 709
 - 16.2.1. *Illustrative Systems of Equations* 709
 - 16.2.2. *Endogeneity and Causality* 712
 - 16.2.3. *A General Notation for Linear Simultaneous Equations Models* 714
 - 16.2.4. *Nonlinear Systems of Equations* 719
- 16.3. The Problem of Identification 719
 - 16.3.1. *The Rank and Order Conditions for Identification* 724
 - 16.3.2. *Identification Through Nonsample Information* 730
- 16.4. Methods of Estimation 734
- 16.5. Single Equation—Limited Information Methods 735
 - 16.5.1. *Ordinary Least Squares* 735
 - 16.5.1a. *Least Squares Estimation of Triangular Systems* 736
 - 16.5.1b. *Indirect Least Squares* 737
 - 16.5.2. *Estimation by Instrumental Variables and GMM* 738
 - 16.5.2a. *Estimating an Exactly Identified Equation* 739
 - 16.5.2b. *Two-Stage Least Squares* 740
 - 16.5.2c. *GMM Estimation* 742
 - 16.5.2d. *Limited Information Maximum Likelihood and the k Class of Estimators* 744
 - 16.5.2e. *Two-Stage Least Squares with Autocorrelation* 748
 - 16.5.2f. *Two-Stage Least Squares in Models That Are Nonlinear in Variables* 750
- 16.6. System Methods of Estimation 751
 - 16.6.1. *Three-Stage Least Squares* 752
 - 16.6.2. *Full-Information Maximum Likelihood* 754
 - 16.6.3. *GMM Estimation* 757
- 16.7. Comparison of Methods 759
- 16.8. Specification Tests 761
- 16.9. Properties of Dynamic Models 764
 - 16.9.1. *Dynamic Models and Their Multipliers* 764
 - 16.9.2. *Stability* 767
 - 16.9.3. *Adjustment to Equilibrium* 770
- Exercises 773
- Appendix 775

CHAPTER 17 Regressions with Lagged Variables 781

- 17.1. Introduction 781
- 17.2. Distributed Lag Models 782
 - 17.2.1. *Lagged Effects in a Regression Model* 783
 - 17.2.2. *The Lag and Difference Operators* 785
 - 17.2.3. *Unrestricted Finite Distributed Lag Models* 786
 - 17.2.4. *Polynomial Distributed Lag Models* 789

17.2.4a. <i>Estimation by Restricted Least Squares</i>	794
17.2.4b. <i>Determining the Degree of the Polynomial</i>	794
17.2.4c. <i>Determining the Lag Length</i>	795
17.2.5. <i>The Geometric Lag Model—A Lagged Dependent Variable</i>	796
17.2.5a. <i>Economic Models with Geometric Lags</i>	797
17.2.5b. <i>Estimating the Autoregressive Form of the Geometric Lag Model</i>	799
17.2.5c. <i>Uncorrelated Disturbances</i>	799
17.2.5d. <i>Autocorrelated Disturbances—Instrumental Variables</i>	800
17.2.5e. <i>Autoregressive Disturbances—Hatanaka's Estimator and the Maximum Likelihood Estimator</i>	800
17.3. Dynamic Regression Models	804
17.3.1. <i>Nonlinear Least Squares Estimation of ARMA and ARMAX Models</i>	806
17.3.2. <i>Computation of the Lag Weights in the ARMAX Model</i>	810
17.3.3. <i>Stability of a Dynamic Equation</i>	812
17.3.4. <i>Forecasting</i>	813
17.4. Vector Autoregressions	815
17.4.1. <i>Testing for Granger Causality</i>	816
17.4.2. <i>Impulse Response Functions</i>	817
17.4.3. <i>Structural VARs</i>	820
17.4.4. <i>VARs in Microeconomics</i>	820
Exercises	821
 CHAPTER 18 Time-Series Models 823	
18.1. Introduction	823
18.2. Stationary Stochastic Processes	824
18.2.1. <i>Autoregressive Moving-Average Processes</i>	825
18.2.2. <i>Stationarity and Invertibility</i>	827
18.2.3. <i>Autocorrelations of a Stationary Stochastic Process</i>	830
18.2.4. <i>Partial Autocorrelations of a Stationary Stochastic Process</i>	833
18.2.5. <i>Modeling Univariate Time Series</i>	835
18.2.6. <i>Estimation of the Parameters of a Univariate Time Series</i>	837
18.3. Nonstationary Processes and Unit Roots	841
18.3.1. <i>Integrated Processes and Differencing</i>	842
18.3.2. <i>Random Walks, Trends, and Spurious Regressions</i>	844
18.3.3. <i>Tests for Unit Roots in Economic Data</i>	847
18.4. Cointegration	851
18.4.1. <i>Common Trends</i>	854
18.4.2. <i>Error Correction and VAR Representations</i>	855
18.4.3. <i>Testing for Cointegration</i>	856
18.4.4. <i>Estimating Cointegration Relationships</i>	859
18.5. Generalized Autoregressive Conditional Heteroscedasticity	859
18.5.1. <i>Maximum Likelihood Estimation of the GARCH Model</i>	861
18.5.1a. <i>Estimating the Variance Parameters</i>	862
18.5.1b. <i>Estimating the Regression Parameters</i>	863

18.5.1c. Computing the Derivatives	865
18.5.1d. Summary	865
18.5.2. Pseudo-Maximum Likelihood Estimation	866
18.5.3. Testing for GARCH Effects	867

Exercise 870

CHAPTER 19 Models with Discrete Dependent Variables 871

19.1. Introduction	871
19.2. Discrete Choice Models	872
19.3. Models for Binary Choice	873
19.3.1. The Regression Approach	873
19.3.2. Index Function and Random Utility Models	880
19.4. Estimation and Inference in Binary Choice Models	882
19.4.1. Specification Tests in Binary Choice Models	888
19.4.1a. Testing for Omitted Variables	889
19.4.1b. Testing for Heteroscedasticity	889
19.4.2. Measuring Goodness of Fit	891
19.4.3. Analysis of Proportions Data	894
19.5. Recent Developments in Binary Choice Modeling	896
19.5.1. Random and Fixed Effects Models for Panel Data	896
19.5.2. Semiparametric Analysis	901
19.5.3. The Maximum Score Estimator (MSCORE)	902
19.5.4. A Kernel Estimator for a Nonparametric Regression Function	904
19.6. Bivariate and Multivariate Probit Models	906
19.6.1. Maximum Likelihood Estimation	907
19.6.2. Extensions	911
19.6.2a. A Multivariate Probit Model	911
19.6.2b. A Model with Censoring	912
19.7. Logit Models for Multiple Choices	912
19.7.1. The Multinomial Logit Model	914
19.7.2. The Conditional Logit Model	917
19.7.3. The Independence of Irrelevant Alternatives	920
19.7.4. Nested Logit Models	921
19.8. Ordered Data	926
19.9. Models for Count Data	931
19.9.1. Measuring Goodness of Fit	935
19.9.2. Censoring and Truncation	936
19.9.3. Testing for Overdispersion	937
19.9.4. Heterogeneity and the Negative Binomial Regression Model	939
19.9.5. Poisson Models for Panel Data	940
19.9.6. Hurdle and Zero-Altered Poisson Models	943
Exercises	946

CHAPTER 20 Limited Dependent Variable and Duration Models 948

20.1. Introduction 948



20.2. Truncation	949
20.2.1. Truncated Distributions	949
20.2.2. Moments of Truncated Distributions	950
20.2.3. The Truncated Regression Model	954
20.2.3a. Least Squares Estimation	956
20.2.3b. Maximum Likelihood Estimation	956
20.3. Censored Data	959
20.3.1. The Censored Normal Distribution	959
20.3.2. The Censored Regression Model—Tobit Analysis	962
20.3.3. Estimation	965
20.3.4. Some Issues in Specification	967
20.3.4a. Heteroscedasticity	967
20.3.4b. Misspecification of Prob{y* < 0}	970
20.3.4c. Nonnormality	971
20.3.4d. Conditional Moment Tests	972
20.4. Selection—Incidental Truncation	974
20.4.1. Incidental Truncation in a Bivariate Distribution	975
20.4.2. Regression in a Model of Selection	975
20.4.3. Estimation	978
20.4.4. Treatment Effects	981
20.4.5. The Normality Assumption	983
20.4.6. Selection in Qualitative Response Models	983
20.5. Models for Duration Data	984
20.5.1. Duration Data	985
20.5.2. A Regression-Like Approach—Parametric Models of Duration	986
20.5.2a. Theoretical Background	986
20.5.2b. Models of the Hazard Rate	987
20.5.2c. Maximum Likelihood Estimation	989
20.5.2d. Exogenous Variables	991
20.5.2e. Specification Analysis	993
20.5.2f. Heterogeneity	995
20.5.3. Other Approaches	997
Exercises	999

APPENDIX TABLES 1001

1. Cumulative Normal Distribution 1001
2. Ordinates of the Standard Normal Density 1002
3. Percentiles of the Student's *t* Distribution 1003
4. Percentiles of the Chi-Squared Distribution 1004
5. 95th Percentile of the *F* Distribution 1006
6. 99th Percentile of the *F* Distribution 1008
- 7a. Durbin-Watson Statistic: 1 Percent Significance Points of *dL* and *dU* 1010

- 7b. Durbin-Watson Statistic: 5 Percent Significance Points of d_L and d_U 1014
- 8. Five Percent Significance Points of $d_{4,L}$ and $d_{4,U}$ for Regressions With Quarterly Dummy Variables ($k = k' + 1$)

REFERENCES 1019

AUTHOR INDEX 1054

SUBJECT INDEX 1058