

FIFTH EDITION
ECONOMETRIC ANALYSIS



William H. Greene

New York University

CONTENTS



Preface xxvii

CHAPTER 1: Introduction 1

- 1.1 Econometrics 1
- 1.2 Econometric Modeling 1
- 1.3 Data and Methodology 4
- 1.4 Plan of the Book 5

CHAPTER 2 The Classical Multiple Linear Regression Model 7

- 2.1 Introduction 7
- 2.2 The Linear Regression Model 7
- 2.3 Assumptions of the Classical Linear Regression Model 10
 - 2.3.1 *Linearity of the Regression Model* 11
 - 2.3.2 *Full Rank* 13
 - 2.3.3 *Regression* 14
 - 2.3.4 *Spherical Disturbances* 15
 - 2.3.5 *Data Generating Process for the Regressors* 16
 - 2.3.6 *Normality* 17
- 2.4 Summary and Conclusions 18

CHAPTER 3 Least Squares 19

- 3.1 Introduction 19
- 3.2 Least Squares Regression 19
 - 3.2.1 *The Least Squares Coefficient Vector* 20
 - 3.2.2 *Application: An Investment Equation* 21
 - 3.2.3 *Algebraic Aspects of The Least Squares Solution* 24
 - 3.2.4 *Projection* 24
- 3.3 Partitioned Regression and Partial Regression 26
- 3.4 Partial Regression and Partial Correlation Coefficients 28
- 3.5 Goodness of Fit and the Analysis of Variance 31
 - 3.5.1 *The Adjusted R-Squared and a Measure of Fit* 34
 - 3.5.2 *R-Squared and the Constant Term in the Model* 36
 - 3.5.3 *Comparing Models* 37
- 3.6 Summary and Conclusions 38

CHAPTER 4 Finite-Sample Properties of the Least Squares Estimator 41

- 4.1 Introduction 41
- 4.2 Motivating Least Squares 42
 - 4.2.1 *The Population Orthogonality Conditions* 42
 - 4.2.2 *Minimum Mean Squared Error Predictor* 43
 - 4.2.3 *Minimum Variance Linear Unbiased Estimation* 44
- 4.3 Unbiased Estimation 44
- 4.4 The Variance of the Least Squares Estimator and the Gauss Markov Theorem 45
- 4.5 The Implications of Stochastic Regressors 47
- 4.6 Estimating the Variance of the Least Squares Estimator 48
- 4.7 The Normality Assumption and Basic Statistical Inference 50
 - 4.7.1 *Testing a Hypothesis About a Coefficient* 50
 - 4.7.2 *Confidence Intervals for Parameters* 52
 - 4.7.3 *Confidence Interval for a Linear Combination of Coefficients: The Oaxaca Decomposition* 53
 - 4.7.4 *Testing the Significance of the Regression* 54
 - 4.7.5 *Marginal Distributions of the Test Statistics* 55
- 4.8 Finite-Sample Properties of Least Squares 55
- 4.9 Data Problems 56
 - 4.9.1 *Multicollinearity* 56
 - 4.9.2 *Missing Observations* 59
 - 4.9.3 *Regression Diagnostics and Influential Data Points* 60
- 4.10 Summary and Conclusions 61

CHAPTER 5 Large-Sample Properties of the Least Squares and Instrumental Variables Estimators 65

- 5.1 Introduction 65
- 5.2 Asymptotic Properties of the Least Squares Estimator 65
 - 5.2.1 *Consistency of the Least Squares Estimator of β* 66
 - 5.2.2 *Asymptotic Normality of the Least Squares Estimator* 67
 - 5.2.3 *Consistency of s^2 and the Estimator of Asy. Var[$\mathbf{b}$]* 69
 - 5.2.4 *Asymptotic Distribution of a Function of $\mathbf{b}$: The Delta Method* 70
 - 5.2.5 *Asymptotic Efficiency* 70
- 5.3 More General Cases 72
 - 5.3.1 *Heterogeneity in the Distributions of $\mathbf{x}_i$* 72
 - 5.3.2 *Dependent Observations* 73
- 5.4 Instrumental Variable and Two Stage Least Squares Estimation 74
- 5.5 Hausman's Specification Test and an Application to Instrumental Variable Estimation 80

5.6	Measurement Error	83
5.6.1	<i>Least Squares Attenuation</i>	84
5.6.2	<i>Instrumental Variables Estimation</i>	86
5.6.3	<i>Proxy Variables</i>	87
5.6.4	<i>Application: Income and Education and a Study of Twins</i>	88
5.7	Summary and Conclusions	90
CHAPTER 6	Inference and Prediction	93
6.1	Introduction	93
6.2	Restrictions and Nested Models	93
6.3	Two Approaches to Testing Hypotheses	95
6.3.1	<i>The F Statistic and the Least Squares Discrepancy</i>	95
6.3.2	<i>The Restricted Least Squares Estimator</i>	99
6.3.3	<i>The Loss of Fit from Restricted Least Squares</i>	101
6.4	Nonnormal Disturbances and Large Sample Tests	104
6.5	Testing Nonlinear Restrictions	108
6.6	Prediction	111
6.7	Summary and Conclusions	114
CHAPTER 7	Functional Form and Structural Change	116
7.1	Introduction	116
7.2	Using Binary Variables	116
7.2.1	<i>Binary Variables in Regression</i>	116
7.2.2	<i>Several Categories</i>	117
7.2.3	<i>Several Groupings</i>	118
7.2.4	<i>Threshold Effects and Categorical Variables</i>	120
7.2.5	<i>Spline Regression</i>	121
7.3	Nonlinearity in the Variables	122
7.3.1	<i>Functional Forms</i>	122
7.3.2	<i>Identifying Nonlinearity</i>	124
7.3.3	<i>Intrinsic Linearity and Identification</i>	127
7.4	Modeling and Testing for a Structural Break	130
7.4.1	<i>Different Parameter Vectors</i>	130
7.4.2	<i>Insufficient Observations</i>	131
7.4.3	<i>Change in a Subset of Coefficients</i>	132
7.4.4	<i>Tests of Structural Break with Unequal Variances</i>	133
7.5	Tests of Model Stability	134
7.5.1	<i>Hansen's Test</i>	134
7.5.2	<i>Recursive Residuals and the CUSUMS Test</i>	135
7.5.3	<i>Predictive Test</i>	137
7.5.4	<i>Unknown Timing of the Structural Break</i>	139
7.6	Summary and Conclusions	144

CHAPTER 8 Specification Analysis and Model Selection 148

- 8.1 Introduction 148
- 8.2 Specification Analysis and Model Building 148
 - 8.2.1 *Bias Caused by Omission of Relevant Variables* 148
 - 8.2.2 *Pretest Estimation* 149
 - 8.2.3 *Inclusion of Irrelevant Variables* 150
 - 8.2.4 *Model Building—A General to Simple Strategy* 151
- 8.3 Choosing Between Nonnested Models 152
 - 8.3.1 *Testing Nonnested Hypotheses* 153
 - 8.3.2 *An Encompassing Model* 154
 - 8.3.3 *Comprehensive Approach—The J Test* 154
 - 8.3.4 *The Cox Test* 155
- 8.4 Model Selection Criteria 159
- 8.5 Summary and Conclusions 160

CHAPTER 9 Nonlinear Regression Models 162

- 9.1 Introduction 162
- 9.2 Nonlinear Regression Models 162
 - 9.2.1 *Assumptions of the Nonlinear Regression Model* 163
 - 9.2.2 *The Orthogonality Condition and the Sum of Squares* 164
 - 9.2.3 *The Linearized Regression* 165
 - 9.2.4 *Large Sample Properties of the Nonlinear Least Squares Estimator* 167
 - 9.2.5 *Computing the Nonlinear Least Squares Estimator* 169
- 9.3 Applications 171
 - 9.3.1 *A Nonlinear Consumption Function* 171
 - 9.3.2 *The Box–Cox Transformation* 173
- 9.4 Hypothesis Testing and Parametric Restrictions 175
 - 9.4.1 *Significance Tests for Restrictions: F and Wald Statistics* 175
 - 9.4.2 *Tests Based on the LM Statistic* 177
 - 9.4.3 *A Specification Test for Nonlinear Regressions: The P_E Test* 178
- 9.5 Alternative Estimators for Nonlinear Regression Models 180
 - 9.5.1 *Nonlinear Instrumental Variables Estimation* 181
 - 9.5.2 *Two-Step Nonlinear Least Squares Estimation* 183
 - 9.5.3 *Two-Step Estimation of a Credit Scoring Model* 186
- 9.6 Summary and Conclusions 189

CHAPTER 10 Nonspherical Disturbances—The Generalized Regression Model 191

- 10.1 Introduction 191
- 10.2 Least Squares and Instrumental Variables Estimation 192
 - 10.2.1 *Finite-Sample Properties of Ordinary Least Squares* 193
 - 10.2.2 *Asymptotic Properties of Least Squares* 194
 - 10.2.3 *Asymptotic Properties of Nonlinear Least Squares* 196

10.2.4	<i>Asymptotic Properties of the Instrumental Variables Estimator</i>	196
10.3	Robust Estimation of Asymptotic Covariance Matrices	198
10.4	Generalized Method of Moments Estimation	201
10.5	Efficient Estimation by Generalized Least Squares	207
10.5.1	<i>Generalized Least Squares (GLS)</i>	207
10.5.2	<i>Feasible Generalized Least Squares</i>	209
10.6	Maximum Likelihood Estimation	211
10.7	Summary and Conclusions	212
CHAPTER 11	Heteroscedasticity	215
11.1	Introduction	215
11.2	Ordinary Least Squares Estimation	216
11.2.1	<i>Inefficiency of Least Squares</i>	217
11.2.2	<i>The Estimated Covariance Matrix of $\mathbf{b}$</i>	217
11.2.3	<i>Estimating the Appropriate Covariance Matrix for Ordinary Least Squares</i>	219
11.3	GMM Estimation of the Heteroscedastic Regression Model	221
11.4	Testing for Heteroscedasticity	222
11.4.1	<i>White's General Test</i>	222
11.4.2	<i>The Goldfeld–Quandt Test</i>	223
11.4.3	<i>The Breusch–Pagan/Godfrey LM Test</i>	223
11.5	Weighted Least Squares When Ω is Known	225
11.6	Estimation When Ω Contains Unknown Parameters	227
11.6.1	<i>Two-Step Estimation</i>	227
11.6.2	<i>Maximum Likelihood Estimation</i>	228
11.6.3	<i>Model Based Tests for Heteroscedasticity</i>	229
11.7	Applications	232
11.7.1	<i>Multiplicative Heteroscedasticity</i>	232
11.7.2	<i>Groupwise Heteroscedasticity</i>	235
11.8	Autoregressive Conditional Heteroscedasticity	238
11.8.1	<i>The ARCH(1) Model</i>	238
11.8.2	<i>ARCH(q), ARCH-in-Mean and Generalized ARCH Models</i>	240
11.8.3	<i>Maximum Likelihood Estimation of the GARCH Model</i>	242
11.8.4	<i>Testing for GARCH Effects</i>	244
11.8.5	<i>Pseudo-Maximum Likelihood Estimation</i>	245
11.9	Summary and Conclusions	246
CHAPTER 12	Serial Correlation	250
12.1	Introduction	250
12.2	The Analysis of Time-Series Data	253
12.3	Disturbance Processes	256

12.3.1	<i>Characteristics of Disturbance Processes</i>	256
12.3.2	<i>AR(1) Disturbances</i>	257
12.4	Some Asymptotic Results for Analyzing Time Series Data	259
12.4.1	<i>Convergence of Moments—The Ergodic Theorem</i>	260
12.4.2	<i>Convergence to Normality—A Central Limit Theorem</i>	262
12.5	Least Squares Estimation	265
12.5.1	<i>Asymptotic Properties of Least Squares</i>	265
12.5.2	<i>Estimating the Variance of the Least Squares Estimator</i>	266
12.6	GMM Estimation	268
12.7	Testing for Autocorrelation	268
12.7.1	<i>Lagrange Multiplier Test</i>	269
12.7.2	<i>Box and Pierce's Test and Ljung's Refinement</i>	269
12.7.3	<i>The Durbin–Watson Test</i>	270
12.7.4	<i>Testing in the Presence of a Lagged Dependent Variables</i>	270
12.7.5	<i>Summary of Testing Procedures</i>	271
12.8	Efficient Estimation When Ω Is Known	271
12.9	Estimation When Ω Is Unknown	273
12.9.1	<i>AR(1) Disturbances</i>	273
12.9.2	<i>AR(2) Disturbances</i>	274
12.9.3	<i>Application: Estimation of a Model with Autocorrelation</i>	274
12.9.4	<i>Estimation with a Lagged Dependent Variable</i>	277
12.10	Common Factors	278
12.11	Forecasting in the Presence of Autocorrelation	279
12.12	Summary and Conclusions	280
✓	CHAPTER 13 Models for Panel Data	283
13.1	Introduction	283
13.2	Panel Data Models	283
13.3	Fixed Effects	287
13.3.1	<i>Testing the Significance of the Group Effects</i>	289
13.3.2	<i>The Within- and Between-Groups Estimators</i>	289
13.3.3	<i>Fixed Time and Group Effects</i>	291
13.3.4	<i>Unbalanced Panels and Fixed Effects</i>	293
13.4	Random Effects	293
13.4.1	<i>Generalized Least Squares</i>	295
13.4.2	<i>Feasible Generalized Least Squares When Σ Is Unknown</i>	296
13.4.3	<i>Testing for Random Effects</i>	298
13.4.4	<i>Hausman's Specification Test for the Random Effects Model</i>	301
13.5	Instrumental Variables Estimation of the Random Effects Model	303
13.6	GMM Estimation of Dynamic Panel Data Models	307
13.7	Nonspherical Disturbances and Robust Covariance Estimation	314
13.7.1	<i>Robust Estimation of the Fixed Effects Model</i>	314

13.7.2	<i>Heteroscedasticity in the Random Effects Model</i>	316
13.7.3	<i>Autocorrelation in Panel Data Models</i>	317
13.8	Random Coefficients Models	318
13.9	Covariance Structures for Pooled Time-Series Cross-Sectional Data	320
13.9.1	<i>Generalized Least Squares Estimation</i>	321
13.9.2	<i>Feasible GLS Estimation</i>	322
13.9.3	<i>Heteroscedasticity and the Classical Model</i>	323
13.9.4	<i>Specification Tests</i>	323
13.9.5	<i>Autocorrelation</i>	324
13.9.6	<i>Maximum Likelihood Estimation</i>	326
13.9.7	<i>Application to Grunfeld's Investment Data</i>	329
13.9.8	<i>Summary</i>	333
13.10	Summary and Conclusions	334
CHAPTER 14 Systems of Regression Equations		339
14.1	Introduction	339
14.2	The Seemingly Unrelated Regressions Model	340
14.2.1	<i>Generalized Least Squares</i>	341
14.2.2	<i>Seemingly Unrelated Regressions with Identical Regressors</i>	343
14.2.3	<i>Feasible Generalized Least Squares</i>	344
14.2.4	<i>Maximum Likelihood Estimation</i>	347
14.2.5	<i>An Application from Financial Econometrics: The Capital Asset Pricing Model</i>	351
14.2.6	<i>Maximum Likelihood Estimation of the Seemingly Unrelated Regressions Model with a Block of Zeros in the Coefficient Matrix</i>	357
14.2.7	<i>Autocorrelation and Heteroscedasticity</i>	360
14.3	Systems of Demand Equations: Singular Systems	362
14.3.1	<i>Cobb-Douglas Cost Function</i>	363
14.3.2	<i>Flexible Functional Forms: The Translog Cost Function</i>	366
14.4	Nonlinear Systems and GMM Estimation	369
14.4.1	<i>GLS Estimation</i>	370
14.4.2	<i>Maximum Likelihood Estimation</i>	371
14.4.3	<i>GMM Estimation</i>	372
14.5	Summary and Conclusions	374
CHAPTER 15 Simultaneous-Equations Models		378
15.1	Introduction	378
15.2	Fundamental Issues in Simultaneous-Equations Models	378
15.2.1	<i>Illustrative Systems of Equations</i>	378
15.2.2	<i>Endogeneity and Causality</i>	381
15.2.3	<i>A General Notation for Linear Simultaneous Equations Models</i>	382
15.3	The Problem of Identification	385

15.3.1	<i>The Rank and Order Conditions for Identification</i>	389
15.3.2	<i>Identification Through Other Nonsample Information</i>	394
15.3.3	<i>Identification Through Covariance Restrictions—The Fully Recursive Model</i>	394
15.4	Methods of Estimation	396
15.5	Single Equation: Limited Information Estimation Methods	396
15.5.1	<i>Ordinary Least Squares</i>	396
15.5.2	<i>Estimation by Instrumental Variables</i>	397
15.5.3	<i>Two-Stage Least Squares</i>	398
15.5.4	<i>GMM Estimation</i>	400
15.5.5	<i>Limited Information Maximum Likelihood and the k Class of Estimators</i>	401
15.5.6	<i>Two-Stage Least Squares in Models That Are Nonlinear in Variables</i>	403
15.6	System Methods of Estimation	404
15.6.1	<i>Three-Stage Least Squares</i>	405
15.6.2	<i>Full-Information Maximum Likelihood</i>	407
15.6.3	<i>GMM Estimation</i>	409
15.6.4	<i>Recursive Systems and Exactly Identified Equations</i>	411
15.7	Comparison of Methods—Klein's Model I	411
15.8	Specification Tests	413
15.9	Properties of Dynamic Models	415
15.9.1	<i>Dynamic Models and Their Multipliers</i>	415
15.9.2	<i>Stability</i>	417
15.9.3	<i>Adjustment to Equilibrium</i>	418
15.10	Summary and Conclusions	421
CHAPTER 16 Estimation Frameworks in Econometrics		425
16.1	Introduction	425
16.2	Parametric Estimation and Inference	427
16.2.1	<i>Classical Likelihood Based Estimation</i>	428
16.2.2	<i>Bayesian Estimation</i>	429
16.2.2.a	<i>Bayesian Analysis of the Classical Regression Model</i>	430
16.2.2.b	<i>Point Estimation</i>	434
16.2.2.c	<i>Interval Estimation</i>	435
16.2.2.d	<i>Estimation with an Informative Prior Density</i>	435
16.2.2.e	<i>Hypothesis Testing</i>	437
16.2.3	<i>Using Bayes Theorem in a Classical Estimation Problem: The Latent Class Model</i>	439
16.2.4	<i>Hierarchical Bayes Estimation of a Random Parameters Model by Markov Chain Monte Carlo Simulation</i>	444
16.3	Semiparametric Estimation	447
16.3.1	<i>GMM Estimation in Econometrics</i>	447
16.3.2	<i>Least Absolute Deviations Estimation</i>	448

16.3.3	<i>Partially Linear Regression</i>	450
16.3.4	<i>Kernel Density Methods</i>	452
16.4	<i>Nonparametric Estimation</i>	453
16.4.1	<i>Kernel Density Estimation</i>	453
16.4.2	<i>Nonparametric Regression</i>	457
16.5	<i>Properties of Estimators</i>	460
16.5.1	<i>Statistical Properties of Estimators</i>	460
16.5.2	<i>Extremum Estimators</i>	461
16.5.3	<i>Assumptions for Asymptotic Properties of Extremum Estimators</i>	461
16.5.4	<i>Asymptotic Properties of Estimators</i>	464
16.5.5	<i>Testing Hypotheses</i>	465
16.6	<i>Summary and Conclusions</i>	466
CHAPTER 17	Maximum Likelihood Estimation	468
17.1	<i>Introduction</i>	468
17.2	<i>The Likelihood Function and Identification of the Parameters</i>	468
17.3	<i>Efficient Estimation: The Principle of Maximum Likelihood</i>	470
17.4	<i>Properties of Maximum Likelihood Estimators</i>	472
17.4.1	<i>Regularity Conditions</i>	473
17.4.2	<i>Properties of Regular Densities</i>	474
17.4.3	<i>The Likelihood Equation</i>	476
17.4.4	<i>The Information Matrix Equality</i>	476
17.4.5	<i>Asymptotic Properties of the Maximum Likelihood Estimator</i>	476
17.4.5.a	<i>Consistency</i>	477
17.4.5.b	<i>Asymptotic Normality</i>	478
17.4.5.c	<i>Asymptotic Efficiency</i>	479
17.4.5.d	<i>Invariance</i>	480
17.4.5.e	<i>Conclusion</i>	480
17.4.6	<i>Estimating the Asymptotic Variance of the Maximum Likelihood Estimator</i>	480
17.4.7	<i>Conditional Likelihoods and Econometric Models</i>	482
17.5	<i>Three Asymptotically Equivalent Test Procedures</i>	484
17.5.1	<i>The Likelihood Ratio Test</i>	484
17.5.2	<i>The Wald Test</i>	486
17.5.3	<i>The Lagrange Multiplier Test</i>	489
17.5.4	<i>An Application of the Likelihood Based Test Procedures</i>	490
17.6	<i>Applications of Maximum Likelihood Estimation</i>	492
17.6.1	<i>The Normal Linear Regression Model</i>	492
17.6.2	<i>Maximum Likelihood Estimation of Nonlinear Regression Models</i>	496
17.6.3	<i>Nonnormal Disturbances—The Stochastic Frontier Model</i>	501
17.6.4	<i>Conditional Moment Tests of Specification</i>	505

17.7	Two-Step Maximum Likelihood Estimation	508
17.8	Maximum Simulated Likelihood Estimation	512
17.9	Pseudo-Maximum Likelihood Estimation and Robust Asymptotic Covariance Matrices	518
17.10	Summary and Conclusions	521
CHAPTER 18 The Generalized Method of Moments		525
18.1	Introduction	525
18.2	Consistent Estimation: The Method of Moments	526
18.2.1	<i>Random Sampling and Estimating the Parameters of Distributions</i>	527
18.2.2	<i>Asymptotic Properties of the Method of Moments Estimator</i>	531
18.2.3	<i>Summary—The Method of Moments</i>	533
18.3	The Generalized Method of Moments (GMM) Estimator	533
18.3.1	<i>Estimation Based on Orthogonality Conditions</i>	534
18.3.2	<i>Generalizing the Method of Moments</i>	536
18.3.3	<i>Properties of the GMM Estimator</i>	540
18.3.4	<i>GMM Estimation of Some Specific Econometric Models</i>	544
18.4	Testing Hypotheses in the GMM Framework	548
18.4.1	<i>Testing the Validity of the Moment Restrictions</i>	548
18.4.2	<i>GMM Counterparts to the Wald, LM, and LR Tests</i>	549
18.5	Application: GMM Estimation of a Dynamic Panel Data Model of Local Government Expenditures	551
18.6	Summary and Conclusions	555
CHAPTER 19 Models with Lagged Variables		558
19.1	Introduction	558
19.2	Dynamic Regression Models	559
19.2.1	<i>Lagged Effects in a Dynamic Model</i>	560
19.2.2	<i>The Lag and Difference Operators</i>	562
19.2.3	<i>Specification Search for the Lag Length</i>	564
19.3	Simple Distributed Lag Models	565
19.3.1	<i>Finite Distributed Lag Models</i>	565
19.3.2	<i>An Infinite Lag Model: The Geometric Lag Model</i>	566
19.4	Autoregressive Distributed Lag Models	571
19.4.1	<i>Estimation of the ARDL Model</i>	572
19.4.2	<i>Computation of the Lag Weights in the ARDL Model</i>	573
19.4.3	<i>Stability of a Dynamic Equation</i>	573
19.4.4	<i>Forecasting</i>	576
19.5	Methodological Issues in the Analysis of Dynamic Models	579
19.5.1	<i>An Error Correction Model</i>	579
19.5.2	<i>Autocorrelation</i>	581

19.5.3	<i>Specification Analysis</i>	582
19.5.4	<i>Common Factor Restrictions</i>	583
19.6	<i>Vector Autoregressions</i>	586
19.6.1	<i>Model Forms</i>	587
19.6.2	<i>Estimation</i>	588
19.6.3	<i>Testing Procedures</i>	589
19.6.4	<i>Exogeneity</i>	590
19.6.5	<i>Testing for Granger Causality</i>	592
19.6.6	<i>Impulse Response Functions</i>	593
19.6.7	<i>Structural VARs</i>	595
19.6.8	<i>Application: Policy Analysis with a VAR</i>	596
19.6.9	<i>VARs in Microeconomics</i>	602
19.7	<i>Summary and Conclusions</i>	605

CHAPTER 20 Time-Series Models 608

20.1	<i>Introduction</i>	608
20.2	<i>Stationary Stochastic Processes</i>	609
20.2.1	<i>Autoregressive Moving-Average Processes</i>	609
20.2.2	<i>Stationarity and Invertibility</i>	611
20.2.3	<i>Autocorrelations of a Stationary Stochastic Process</i>	614
20.2.4	<i>Partial Autocorrelations of a Stationary Stochastic Process</i>	617
20.2.5	<i>Modeling Univariate Time Series</i>	619
20.2.6	<i>Estimation of the Parameters of a Univariate Time Series</i>	621
20.2.7	<i>The Frequency Domain</i>	624
20.2.7.a	<i>Theoretical Results</i>	625
20.2.7.b	<i>Empirical Counterparts</i>	627
20.3	<i>Nonstationary Processes and Unit Roots</i>	631
20.3.1	<i>Integrated Processes and Differencing</i>	631
20.3.2	<i>Random Walks, Trends, and Spurious Regressions</i>	632
20.3.3	<i>Tests for Unit Roots in Economic Data</i>	636
20.3.4	<i>The Dickey–Fuller Tests</i>	637
20.3.5	<i>Long Memory Models</i>	647
20.4	<i>Cointegration</i>	649
20.4.1	<i>Common Trends</i>	653
20.4.2	<i>Error Correction and VAR Representations</i>	654
20.4.3	<i>Testing for Cointegration</i>	655
20.4.4	<i>Estimating Cointegration Relationships</i>	657
20.4.5	<i>Application: German Money Demand</i>	657
20.4.5.a	<i>Cointegration Analysis and a Long Run Theoretical Model</i>	659
20.4.5.b	<i>Testing for Model Instability</i>	659
20.5	<i>Summary and Conclusions</i>	660

CHAPTER 21	Models for Discrete Choice	663
21.1	Introduction	663
21.2	Discrete Choice Models	663
21.3	Models for Binary Choice	665
21.3.1	<i>The Regression Approach</i>	665
21.3.2	<i>Latent Regression—Index Function Models</i>	668
21.3.3	<i>Random Utility Models</i>	670
21.4	Estimation and Inference in Binary Choice Models	670
21.4.1	<i>Robust Covariance Matrix Estimation</i>	673
21.4.2	<i>Marginal Effects</i>	674
21.4.3	<i>Hypothesis Tests</i>	676
21.4.4	<i>Specification Tests for Binary Choice Models</i>	679
21.4.4.a	<i>Omitted Variables</i>	680
21.4.4.b	<i>Heteroscedasticity</i>	680
21.4.4.c	<i>A Specification Test for Nonnested Models—Testing for the Distribution</i>	682
21.4.5	<i>Measuring Goodness of Fit</i>	683
21.4.6	<i>Analysis of Proportions Data</i>	686
21.5	Extensions of the Binary Choice Model	689
21.5.1	<i>Random and Fixed Effects Models for Panel Data</i>	689
21.5.1.a	<i>Random Effects Models</i>	690
21.5.1.b	<i>Fixed Effects Models</i>	695
21.5.2	<i>Semiparametric Analysis</i>	700
21.5.3	<i>The Maximum Score Estimator (MSCORE)</i>	702
21.5.4	<i>Semiparametric Estimation</i>	704
21.5.5	<i>A Kernel Estimator for a Nonparametric Regression Function</i>	706
21.5.6	<i>Dynamic Binary Choice Models</i>	708
21.6	Bivariate and Multivariate Probit Models	710
21.6.1	<i>Maximum Likelihood Estimation</i>	710
21.6.2	<i>Testing for Zero Correlation</i>	712
21.6.3	<i>Marginal Effects</i>	712
21.6.4	<i>Sample Selection</i>	713
21.6.5	<i>A Multivariate Probit Model</i>	714
21.6.6	<i>Application: Gender Economics Courses in Liberal Arts Colleges</i>	715
21.7	Logit Models for Multiple Choices	719
21.7.1	<i>The Multinomial Logit Model</i>	720
21.7.2	<i>The Conditional Logit Model</i>	723
21.7.3	<i>The Independence from Irrelevant Alternatives</i>	724
21.7.4	<i>Nested Logit Models</i>	725
21.7.5	<i>A Heteroscedastic Logit Model</i>	727
21.7.6	<i>Multinomial Models Based on the Normal Distribution</i>	727
21.7.7	<i>A Random Parameters Model</i>	728

21.7.8	<i>Application: Conditional Logit Model for Travel Mode Choice</i>	729
21.8	Ordered Data	736
21.9	Models for Count Data	740
21.9.1	<i>Measuring Goodness of Fit</i>	741
21.9.2	<i>Testing for Overdispersion</i>	743
21.9.3	<i>Heterogeneity and the Negative Binomial Regression Model</i>	744
21.9.4	<i>Application: The Poisson Regression Model</i>	745
21.9.5	<i>Poisson Models for Panel Data</i>	747
21.9.6	<i>Hurdle and Zero-Altered Poisson Models</i>	749
21.10	Summary and Conclusions	752
CHAPTER 22	Limited Dependent Variable and Duration Models	756
22.1	Introduction	756
22.2	Truncation	756
22.2.1	<i>Truncated Distributions</i>	757
22.2.2	<i>Moments of Truncated Distributions</i>	758
22.2.3	<i>The Truncated Regression Model</i>	760
22.3	Censored Data	761
22.3.1	<i>The Censored Normal Distribution</i>	762
22.3.2	<i>The Censored Regression (Tobit) Model</i>	764
22.3.3	<i>Estimation</i>	766
22.3.4	<i>Some Issues in Specification</i>	768
22.3.4.a	<i>Heteroscedasticity</i>	768
22.3.4.b	<i>Misspecification of Prob[y* < 0]</i>	770
22.3.4.c	<i>Nonnormality</i>	771
22.3.4.d	<i>Conditional Moment Tests</i>	772
22.3.5	<i>Censoring and Truncation in Models for Counts</i>	773
22.3.6	<i>Application: Censoring in the Tobit and Poisson Regression Models</i>	774
22.4	The Sample Selection Model	780
22.4.1	<i>Incidental Truncation in a Bivariate Distribution</i>	781
22.4.2	<i>Regression in a Model of Selection</i>	782
22.4.3	<i>Estimation</i>	784
22.4.4	<i>Treatment Effects</i>	787
22.4.5	<i>The Normality Assumption</i>	789
22.4.6	<i>Selection in Qualitative Response Models</i>	790
22.5	Models for Duration Data	790
22.5.1	<i>Duration Data</i>	791
22.5.2	<i>A Regression-Like Approach: Parametric Models of Duration</i>	792
22.5.2.a	<i>Theoretical Background</i>	792
22.5.2.b	<i>Models of the Hazard Function</i>	793
22.5.2.c	<i>Maximum Likelihood Estimation</i>	794

22.5.2.d	<i>Exogenous Variables</i>	796
22.5.2.e	<i>Heterogeneity</i>	797
22.5.3	<i>Other Approaches</i>	798
22.6	<i>Summary and Conclusions</i>	801

APPENDIX A Matrix Algebra 803

A.1	<i>Terminology</i>	803
A.2	<i>Algebraic Manipulation of Matrices</i>	803
A.2.1	<i>Equality of Matrices</i>	803
A.2.2	<i>Transposition</i>	804
A.2.3	<i>Matrix Addition</i>	804
A.2.4	<i>Vector Multiplication</i>	805
A.2.5	<i>A Notation for Rows and Columns of a Matrix</i>	805
A.2.6	<i>Matrix Multiplication and Scalar Multiplication</i>	805
A.2.7	<i>Sums of Values</i>	807
A.2.8	<i>A Useful Idempotent Matrix</i>	808
A.3	<i>Geometry of Matrices</i>	809
A.3.1	<i>Vector Spaces</i>	809
A.3.2	<i>Linear Combinations of Vectors and Basis Vectors</i>	811
A.3.3	<i>Linear Dependence</i>	811
A.3.4	<i>Subspaces</i>	813
A.3.5	<i>Rank of a Matrix</i>	814
A.3.6	<i>Determinant of a Matrix</i>	816
A.3.7	<i>A Least Squares Problem</i>	817
A.4	<i>Solution of a System of Linear Equations</i>	819
A.4.1	<i>Systems of Linear Equations</i>	819
A.4.2	<i>Inverse Matrices</i>	820
A.4.3	<i>Nonhomogeneous Systems of Equations</i>	822
A.4.4	<i>Solving the Least Squares Problem</i>	822
A.5	<i>Partitioned Matrices</i>	822
A.5.1	<i>Addition and Multiplication of Partitioned Matrices</i>	823
A.5.2	<i>Determinants of Partitioned Matrices</i>	823
A.5.3	<i>Inverses of Partitioned Matrices</i>	823
A.5.4	<i>Deviations from Means</i>	824
A.5.5	<i>Kronecker Products</i>	824
A.6	<i>Characteristic Roots and Vectors</i>	825
A.6.1	<i>The Characteristic Equation</i>	825
A.6.2	<i>Characteristic Vectors</i>	826
A.6.3	<i>General Results for Characteristic Roots and Vectors</i>	826
A.6.4	<i>Diagonalization and Spectral Decomposition of a Matrix</i>	827
A.6.5	<i>Rank of a Matrix</i>	827
A.6.6	<i>Condition Number of a Matrix</i>	829
A.6.7	<i>Trace of a Matrix</i>	829
A.6.8	<i>Determinant of a Matrix</i>	830
A.6.9	<i>Powers of a Matrix</i>	830

A.6.10	<i>Idempotent Matrices</i>	832
A.6.11	<i>Factoring a Matrix</i>	832
A.6.12	<i>The Generalized Inverse of a Matrix</i>	833
A.7	<i>Quadratic Forms and Definite Matrices</i>	834
A.7.1	<i>Nonnegative Definite Matrices</i>	835
A.7.2	<i>Idempotent Quadratic Forms</i>	836
A.7.3	<i>Comparing Matrices</i>	836
A.8	<i>Calculus and Matrix Algebra</i>	837
A.8.1	<i>Differentiation and the Taylor Series</i>	837
A.8.2	<i>Optimization</i>	840
A.8.3	<i>Constrained Optimization</i>	842
A.8.4	<i>Transformations</i>	844
APPENDIX B	Probability and Distribution Theory	845
B.1	<i>Introduction</i>	845
B.2	<i>Random Variables</i>	845
B.2.1	<i>Probability Distributions</i>	845
B.2.2	<i>Cumulative Distribution Function</i>	846
B.3	<i>Expectations of a Random Variable</i>	847
B.4	<i>Some Specific Probability Distributions</i>	849
B.4.1	<i>The Normal Distribution</i>	849
B.4.2	<i>The Chi-Squared, t, and F Distributions</i>	851
B.4.3	<i>Distributions With Large Degrees of Freedom</i>	853
B.4.4	<i>Size Distributions: The Lognormal Distribution</i>	854
B.4.5	<i>The Gamma and Exponential Distributions</i>	855
B.4.6	<i>The Beta Distribution</i>	855
B.4.7	<i>The Logistic Distribution</i>	855
B.4.8	<i>Discrete Random Variables</i>	855
B.5	<i>The Distribution of a Function of a Random Variable</i>	856
B.6	<i>Representations of a Probability Distribution</i>	858
B.7	<i>Joint Distributions</i>	860
B.7.1	<i>Marginal Distributions</i>	860
B.7.2	<i>Expectations in a Joint Distribution</i>	861
B.7.3	<i>Covariance and Correlation</i>	861
B.7.4	<i>Distribution of a Function of Bivariate Random Variables</i>	862
B.8	<i>Conditioning in a Bivariate Distribution</i>	864
B.8.1	<i>Regression: The Conditional Mean</i>	864
B.8.2	<i>Conditional Variance</i>	865
B.8.3	<i>Relationships Among Marginal and Conditional Moments</i>	865
B.8.4	<i>The Analysis of Variance</i>	867
B.9	<i>The Bivariate Normal Distribution</i>	867
B.10	<i>Multivariate Distributions</i>	868
B.10.1	<i>Moments</i>	868

B.10.2	<i>Sets of Linear Functions</i>	869
B.10.3	<i>Nonlinear Functions</i>	870
B.11	The Multivariate Normal Distribution	871
B.11.1	<i>Marginal and Conditional Normal Distributions</i>	871
B.11.2	<i>The Classical Normal Linear Regression Model</i>	872
B.11.3	<i>Linear Functions of a Normal Vector</i>	873
B.11.4	<i>Quadratic Forms in a Standard Normal Vector</i>	873
B.11.5	<i>The F Distribution</i>	875
B.11.6	<i>A Full Rank Quadratic Form</i>	875
B.11.7	<i>Independence of a Linear and a Quadratic Form</i>	876
APPENDIX C Estimation and Inference		877
C.1	Introduction	877
C.2	Samples and Random Sampling	878
C.3	Descriptive Statistics	878
C.4	Statistics as Estimators—Sampling Distributions	882
C.5	Point Estimation of Parameters	885
C.5.1	<i>Estimation in a Finite Sample</i>	885
C.5.2	<i>Efficient Unbiased Estimation</i>	888
C.6	Interval Estimation	890
C.7	Hypothesis Testing	892
C.7.1	<i>Classical Testing Procedures</i>	892
C.7.2	<i>Tests Based on Confidence Intervals</i>	895
C.7.3	<i>Specification Tests</i>	896
APPENDIX D Large Sample Distribution Theory		896
D.1	Introduction	896
D.2	Large-Sample Distribution Theory	897
D.2.1	<i>Convergence in Probability</i>	897
D.2.2	<i>Other Forms of Convergence and Laws of Large Numbers</i>	900
D.2.3	<i>Convergence of Functions</i>	903
D.2.4	<i>Convergence to a Random Variable</i>	904
D.2.5	<i>Convergence in Distribution: Limiting Distributions</i>	906
D.2.6	<i>Central Limit Theorems</i>	908
D.2.7	<i>The Delta Method</i>	913
D.3	Asymptotic Distributions	914
D.3.1	<i>Asymptotic Distribution of a Nonlinear Function</i>	916
D.3.2	<i>Asymptotic Expectations</i>	917
D.4	Sequences and the Order of a Sequence	918
APPENDIX E Computation and Optimization		919
E.1	Introduction	919
E.2	Data Input and Generation	920
E.2.1	<i>Generating Pseudo-Random Numbers</i>	920

/ E.2.2	<i>Sampling from a Standard Uniform Population</i>	921
E.2.3	<i>Sampling from Continuous Distributions</i>	921
E.2.4	<i>Sampling from a Multivariate Normal Population</i>	922
E.2.5	<i>Sampling from a Discrete Population</i>	922
E.2.6	<i>The Gibbs Sampler</i>	922
E.3	Monte Carlo Studies	923
E.4	Bootstrapping and the Jackknife	924
E.5	Computation in Econometrics	925
E.5.1	<i>Computing Integrals</i>	926
E.5.2	<i>The Standard Normal Cumulative Distribution Function</i>	926
E.5.3	<i>The Gamma and Related Functions</i>	927
E.5.4	<i>Approximating Integrals by Quadrature</i>	928
E.5.5	<i>Monte Carlo Integration</i>	929
E.5.6	<i>Multivariate Normal Probabilities and Simulated Moments</i>	931
E.5.7	<i>Computing Derivatives</i>	933
E.6	Optimization	933
E.6.1	<i>Algorithms</i>	935
E.6.2	<i>Gradient Methods</i>	935
E.6.3	<i>Aspects of Maximum Likelihood Estimation</i>	939
E.6.4	<i>Optimization with Constraints</i>	941
E.6.5	<i>Some Practical Considerations</i>	942
E.6.6	<i>Examples</i>	943
APPENDIX F	Data Sets Used in Applications	946
APPENDIX G	Statistical Tables	953
References	959	
Author Index	995	
Subject Index	1002	