

Morphometric tools for landmark data

Geometry and biology

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6.2 Geometric version

The same pair of directions may be constructed from two pairs of shape coordinates by ruler and compass. For small changes in the shape-coordinate plane, ordinary Euclidean distance is proportional to log anisotropy, and the principal directions can be approximated by an angle bisection.

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6.4 Analyses of more than three landmarks

The transect theorem is explained: for any two mean configurations of landmarks, the homologous distance measures showing the greatest and least ratios of change are transects of triangles. But the ratios of simple interlandmark distances are not sufficient to produce these extrema; this failure accounts for the inefficiency of many of the distance-based approaches.

6.5 Biometric analysis of triangles of landmarks: examples

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6.6 A comment on "finite elements"

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