

Quantum Computation and Quantum Information

10th Anniversary Edition

Michael A. Nielsen & Isaac L. Chuang

CAMBRIDGE
UNIVERSITY PRESS

Contents

Introduction to the Tenth Anniversary Edition	<i>page</i> xvii
Afterword to the Tenth Anniversary Edition	xix
Preface	xxi
Acknowledgements	xxvii
Nomenclature and notation	xxix
Part I Fundamental concepts	1
1 Introduction and overview	1
1.1 Global perspectives	1
1.1.1 History of quantum computation and quantum information	2
1.1.2 Future directions	12
1.2 Quantum bits	13
1.2.1 Multiple qubits	16
1.3 Quantum computation	17
1.3.1 Single qubit gates	17
1.3.2 Multiple qubit gates	20
1.3.3 Measurements in bases other than the computational basis	22
1.3.4 Quantum circuits	22
1.3.5 Qubit copying circuit?	24
1.3.6 Example: Bell states	25
1.3.7 Example: quantum teleportation	26
1.4 Quantum algorithms	28
1.4.1 Classical computations on a quantum computer	29
1.4.2 Quantum parallelism	30
1.4.3 Deutsch's algorithm	32
1.4.4 The Deutsch-Jozsa algorithm	34
1.4.5 Quantum algorithms summarized	36
1.5 Experimental quantum information processing	42
1.5.1 The Stern-Gerlach experiment	43
1.5.2 Prospects for practical quantum information processing	46
1.6 Quantum information	50
1.6.1 Quantum information theory: example problems	52
1.6.2 Quantum information in a wider context	58

2 Introduction to quantum mechanics	60
2.1 Linear algebra	61
2.1.1 Bases and linear independence	62
2.1.2 Linear operators and matrices	63
2.1.3 The Pauli matrices	65
2.1.4 Inner products	65
2.1.5 Eigenvectors and eigenvalues	68
2.1.6 Adjoints and Hermitian operators	69
2.1.7 Tensor products	71
2.1.8 Operator functions	75
2.1.9 The commutator and anti-commutator	76
2.1.10 The polar and singular value decompositions	78
2.2 The postulates of quantum mechanics	80
2.2.1 State space	80
2.2.2 Evolution	81
2.2.3 Quantum measurement	84
2.2.4 Distinguishing quantum states	86
2.2.5 Projective measurements	87
2.2.6 POVM measurements	90
2.2.7 Phase	93
2.2.8 Composite systems	93
2.2.9 Quantum mechanics: a global view	96
2.3 Application: superdense coding	97
2.4 The density operator	98
2.4.1 Ensembles of quantum states	99
2.4.2 General properties of the density operator	101
2.4.3 The reduced density operator	105
2.5 The Schmidt decomposition and purifications	109
2.6 EPR and the Bell inequality	111
 3 Introduction to computer science	 120
3.1 Models for computation	122
3.1.1 Turing machines	122
3.1.2 Circuits	129
3.2 The analysis of computational problems	135
3.2.1 How to quantify computational resources	136
3.2.2 Computational complexity	138
3.2.3 Decision problems and the complexity classes P and NP	141
3.2.4 A plethora of complexity classes	150
3.2.5 Energy and computation	153
3.3 Perspectives on computer science	161
 Part II Quantum computation	 171
 4 Quantum circuits	 171
4.1 Quantum algorithms	172
4.2 Single qubit operations	174

4.3	Controlled operations	177
4.4	Measurement	185
4.5	Universal quantum gates	188
4.5.1	Two-level unitary gates are universal	189
4.5.2	Single qubit and CNOT gates are universal	191
4.5.3	A discrete set of universal operations	194
4.5.4	Approximating arbitrary unitary gates is generically hard	198
4.5.5	Quantum computational complexity	200
4.6	Summary of the quantum circuit model of computation	202
4.7	Simulation of quantum systems	204
4.7.1	Simulation in action	204
4.7.2	The quantum simulation algorithm	206
4.7.3	An illustrative example	209
4.7.4	Perspectives on quantum simulation	211
5	The quantum Fourier transform and its applications	216
5.1	The quantum Fourier transform	217
5.2	Phase estimation	221
5.2.1	Performance and requirements	223
5.3	Applications: order-finding and factoring	226
5.3.1	Application: order-finding	226
5.3.2	Application: factoring	232
5.4	General applications of the quantum Fourier transform	234
5.4.1	Period-finding	236
5.4.2	Discrete logarithms	238
5.4.3	The hidden subgroup problem	240
5.4.4	Other quantum algorithms?	242
6	Quantum search algorithms	248
6.1	The quantum search algorithm	248
6.1.1	The oracle	248
6.1.2	The procedure	250
6.1.3	Geometric visualization	252
6.1.4	Performance	253
6.2	Quantum search as a quantum simulation	255
6.3	Quantum counting	261
6.4	Speeding up the solution of NP -complete problems	263
6.5	Quantum search of an unstructured database	265
6.6	Optimality of the search algorithm	269
6.7	Black box algorithm limits	271
7	Quantum computers: physical realization	277
7.1	Guiding principles	277
7.2	Conditions for quantum computation	279
7.2.1	Representation of quantum information	279
7.2.2	Performance of unitary transformations	281

7.2.3	Preparation of fiducial initial states	281
7.2.4	Measurement of output result	282
7.3	Harmonic oscillator quantum computer	283
7.3.1	Physical apparatus	283
7.3.2	The Hamiltonian	284
7.3.3	Quantum computation	286
7.3.4	Drawbacks	286
7.4	Optical photon quantum computer	287
7.4.1	Physical apparatus	287
7.4.2	Quantum computation	290
7.4.3	Drawbacks	296
7.5	Optical cavity quantum electrodynamics	297
7.5.1	Physical apparatus	298
7.5.2	The Hamiltonian	300
7.5.3	Single-photon single-atom absorption and refraction	303
7.5.4	Quantum computation	306
7.6	Ion traps	309
7.6.1	Physical apparatus	309
7.6.2	The Hamiltonian	317
7.6.3	Quantum computation	319
7.6.4	Experiment	321
7.7	Nuclear magnetic resonance	324
7.7.1	Physical apparatus	325
7.7.2	The Hamiltonian	326
7.7.3	Quantum computation	331
7.7.4	Experiment	336
7.8	Other implementation schemes	343
Part III	Quantum information	353
8	Quantum noise and quantum operations	353
8.1	Classical noise and Markov processes	354
8.2	Quantum operations	356
8.2.1	Overview	356
8.2.2	Environments and quantum operations	357
8.2.3	Operator-sum representation	360
8.2.4	Axiomatic approach to quantum operations	366
8.3	Examples of quantum noise and quantum operations	373
8.3.1	Trace and partial trace	374
8.3.2	Geometric picture of single qubit quantum operations	374
8.3.3	Bit flip and phase flip channels	376
8.3.4	Depolarizing channel	378
8.3.5	Amplitude damping	380
8.3.6	Phase damping	383

8.4 Applications of quantum operations	386
8.4.1 Master equations	386
8.4.2 Quantum process tomography	389
8.5 Limitations of the quantum operations formalism	394
9 Distance measures for quantum information	399
9.1 Distance measures for classical information	399
9.2 How close are two quantum states?	403
9.2.1 Trace distance	403
9.2.2 Fidelity	409
9.2.3 Relationships between distance measures	415
9.3 How well does a quantum channel preserve information?	416
10 Quantum error-correction	425
10.1 Introduction	426
10.1.1 The three qubit bit flip code	427
10.1.2 Three qubit phase flip code	430
10.2 The Shor code	432
10.3 Theory of quantum error-correction	435
10.3.1 Discretization of the errors	438
10.3.2 Independent error models	441
10.3.3 Degenerate codes	444
10.3.4 The quantum Hamming bound	444
10.4 Constructing quantum codes	445
10.4.1 Classical linear codes	445
10.4.2 Calderbank-Shor-Steane codes	450
10.5 Stabilizer codes	453
10.5.1 The stabilizer formalism	454
10.5.2 Unitary gates and the stabilizer formalism	459
10.5.3 Measurement in the stabilizer formalism	463
10.5.4 The Gottesman-Knill theorem	464
10.5.5 Stabilizer code constructions	464
10.5.6 Examples	467
10.5.7 Standard form for a stabilizer code	470
10.5.8 Quantum circuits for encoding, decoding, and correction	472
10.6 Fault-tolerant quantum computation	474
10.6.1 Fault-tolerance: the big picture	475
10.6.2 Fault-tolerant quantum logic	482
10.6.3 Fault-tolerant measurement	489
10.6.4 Elements of resilient quantum computation	493
11 Entropy and information	500
11.1 Shannon entropy	500
11.2 Basic properties of entropy	502
11.2.1 The binary entropy	502
11.2.2 The relative entropy	504

11.2.3	Conditional entropy and mutual information	505
11.2.4	The data processing inequality	509
11.3	Von Neumann entropy	510
11.3.1	Quantum relative entropy	511
11.3.2	Basic properties of entropy	513
11.3.3	Measurements and entropy	514
11.3.4	Subadditivity	515
11.3.5	Concavity of the entropy	516
11.3.6	The entropy of a mixture of quantum states	518
11.4	Strong subadditivity	519
11.4.1	Proof of strong subadditivity	519
11.4.2	Strong subadditivity: elementary applications	522
12	Quantum information theory	528
12.1	Distinguishing quantum states and the accessible information	529
12.1.1	The Holevo bound	531
12.1.2	Example applications of the Holevo bound	534
12.2	Data compression	536
12.2.1	Shannon's noiseless channel coding theorem	537
12.2.2	Schumacher's quantum noiseless channel coding theorem	542
12.3	Classical information over noisy quantum channels	546
12.3.1	Communication over noisy classical channels	548
12.3.2	Communication over noisy quantum channels	554
12.4	Quantum information over noisy quantum channels	561
12.4.1	Entropy exchange and the quantum Fano inequality	561
12.4.2	The quantum data processing inequality	564
12.4.3	Quantum Singleton bound	568
12.4.4	Quantum error-correction, refrigeration and Maxwell's demon	569
12.5	Entanglement as a physical resource	571
12.5.1	Transforming bi-partite pure state entanglement	573
12.5.2	Entanglement distillation and dilution	578
12.5.3	Entanglement distillation and quantum error-correction	580
12.6	Quantum cryptography	582
12.6.1	Private key cryptography	582
12.6.2	Privacy amplification and information reconciliation	584
12.6.3	Quantum key distribution	586
12.6.4	Privacy and coherent information	592
12.6.5	The security of quantum key distribution	593
	Appendices	608
	Appendix 1: Notes on basic probability theory	608
	Appendix 2: Group theory	610
A2.1	Basic definitions	610
A2.1.1	Generators	611
A2.1.2	Cyclic groups	611
A2.1.3	Cosets	612

A2.2	Representations	612
A2.2.1	Equivalence and reducibility	612
A2.2.2	Orthogonality	613
<i>hi 23</i>	The regular representation	614
A2.3	Fourier transforms	615
Appendix 3: The Solovay—Kitaev theorem		617
Appendix 4: Number theory		625
A4.1	Fundamentals	625
A4.2	Modular arithmetic and Euclid's algorithm	626
A4.3	Reduction of factoring to order-finding	633
A4.4	Continued fractions	635
Appendix 5: Public key cryptography and the RSA cryptosystem		640
Appendix 6: Proof of Lieb's theorem		645
Bibliography		649
Index		665