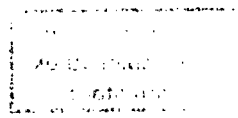
Gregory L. Naber

Topology, Geometry, and Gauge Fields

Foundations

Second Edition



With 65 Illustrations

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