

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

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*Polynomials with Special Regard to  
Reducibility*

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A. SCHINZEL

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