

# *Applied Optimal Control*

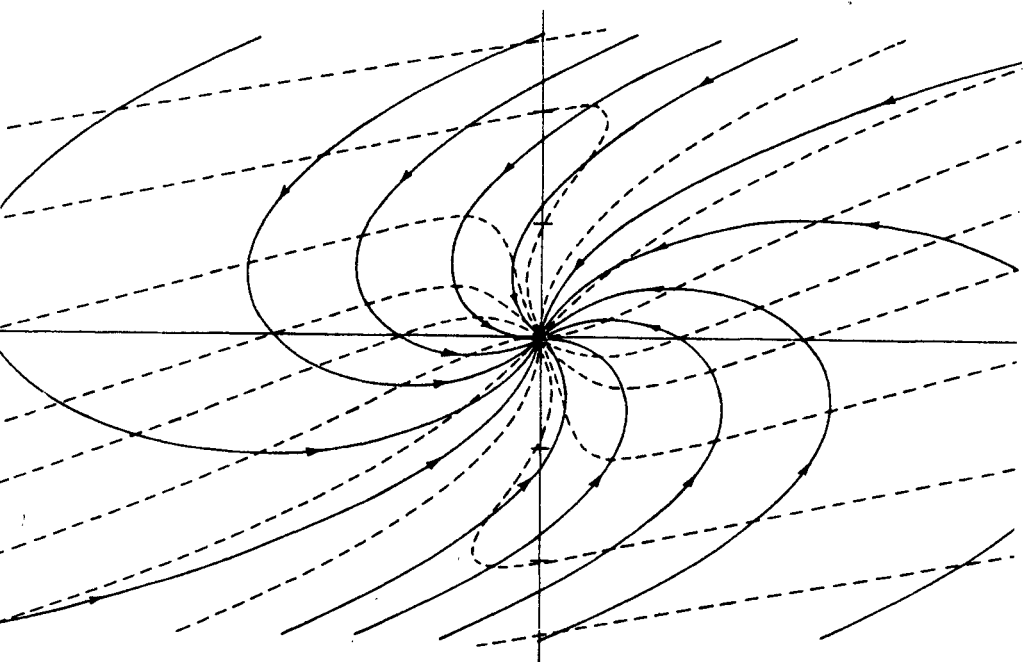
OPTIMIZATION, ESTIMATION, AND CONTROL

**Arthur E. Bryson, Jr.**

*Stanford University*

**Yu-Chi Ho**

*Harvard University*



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LOGICAL DEPENDENCE OF CHAPTERS

