

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

***Computation with finitely
presented groups***

CHARLES C. SIMS

Rutgers University



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