

Lecture Notes in Economics and Mathematical Systems

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Robust Methods and Asymptotic Theory in Nonlinear Econometrics



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