

AUTOMATA, LANGUAGES, AND MACHINES

VOLUME B

Samuel Eilenberg

COLUMBIA UNIVERSITY
NEW YORK

With two chapters by

Bret Tilson

CITY UNIVERSITY OF NEW YORK
QUEENS COLLEGE
NEW YORK



ACADEMIC PRESS New York San Francisco London 1976

A Subsidiary of Harcourt Brace Jovanovich, Publishers

Technische Hochschule Darmstadt	
FACHBEREICH INFORMATIK	
<u>B I B L I O T H E K</u>	
Inventar-Nr.:	9170
Sachgebiete:	F.4
Standort:	

Contents

PREFACE	xi
-------------------	----

CHAPTER I

Transformation Semigroups

1. Semigroups, Monoids, and Groups	1
2. Transformation Semigroups	3
3. Examples of Transformation Semigroups	5
4. Coverings	8
5. Coverings of Semigroups	12
6. Inclusions and Restrictions	14
7. Isomorphisms and Equivalences	16
8. Join, Sum, and Direct Product	18
9. Some Simple Inequalities	22
10. The Wreath Product	26
References	32

CHAPTER II

Decomposition Theorems

1. Decompositions	33
2. Decomposition of Groups	34
3. Some Useful Decompositions	36
4. The Krohn–Rhodes Decomposition	39
5. Comments on the Proof	43
6. Height, Pavings, and Holonomy	43
7. The Holonomy Decomposition Theorem	46
8. Proof of Proposition 7.3	48
9. Examples	51
References	57

CHAPTER III

Transformation Semigroups (continued)

1. Classes and Closed Classes	59
2. Sinks in a ts	62
3. Transitivity Classes	66
4. Idempotents in Semigroups	67
5. Idempotents in a ts	69
6. Localization	72
7. Closed Classes Containing \bar{Z}^*	74
8. The Derived ts and the Trace of a Covering	76
9. The Delay Covering	80
References	86

CHAPTER IV

Primes

1. The Exclusion Operator	87
2. Primes	88
3. Proof of Theorem 2.1	90
4. The Low Primes	94
5. The Primes C and C^*	96
6. The Primes F , \bar{Z} , F^* , and \bar{Z}^*	100
7. Switching Rules	102
8. Summary and Open Problems	105
References	108

CHAPTER V

Semigroups and Varieties

1. Varieties of Semigroups and Monoids	109
2. Varieties Defined by Equations	112
3. Examples of Ultimately Equational Varieties	116
4. Semidirect Products	123
5. Varieties $\mathbf{V} * \mathbf{W}$	129
6. Varieties vs. Weakly Closed Classes	132
7. Closed Varieties	135
8. Examples of Closed Varieties	138
9. Triple Products	142
10. \mathbf{G} -Varieties	144
11. Primes	150
12. A Tabulation	152
References	156

CHAPTER VI

Decomposition of Sequential Functions

1. Syntactic Invariants of Sequential Functions	157
2. Composition	162
3. Decomposition	163
4. Parallel Composition	168
5. Examples of Decompositions	174
6. The Function \mathcal{S}	178
7. Varieties of Sequential Functions	181

CHAPTER VII

Varieties of Sets

1. Syntactic Semigroups	185
2. Syntactic Semigroups and Recognizable Sets	188
3. Varieties of Sets	192
4. Proof of Theorems 3.2 and 3.2s	197
5. Operations on Varieties	199
6. The Syntactic tm and ts of a Set	202

CHAPTER VIII

Examples of Varieties of Sets

1. General Comments	207
2. Finite and Cofinite Sets	208
3. Finitely Generated Varieties	210
4. The Variety \mathbf{D}	214
5. The Variety $\hat{\mathbf{D}}$	216
6. Locally Testable Sets	218
7. A Theorem on Graphs	222
8. Proof of Theorem 6.5	228
9. The $*$ -Variety \mathcal{F}	232
10. p -Groups	238
References	245

CHAPTER IX

Aperiodicity

1. Recognizable Sets and Sequential Functions	247
2. The Concatenation Product	249
3. Schützenberger's Theorem	253

4. The Brzozowski Hierarchy	256
5. $\mathcal{B}_{n,1}$ Are $+$ -Varieties	259
6. The Variety \mathbf{B}_2	261
7. The Variety \mathbf{A}_1	263
References	268

CHAPTER X

Unitary-Prefix Decompositions

1. Unitary-Prefix Decompositions	269
2. A Decomposition	272
3. Two Examples	274
4. Iterated Decomposition	277
5. Periods of Monoids	279
6. Proof of Theorem 5.2	282
References	285

CHAPTER XI

Depth Decomposition Theorem

by Bret Tilson

1. Basic Orderings in Semigroups	287
2. The Depth Decomposition Theorem	295
3. The Rees Matrix Semigroup	297
4. The Reduction Theorem	300
5. Proof of Proposition 2.2	304
6. Comparison with Holonomy Decomposition	308
References	311

CHAPTER XII

Complexity of Semigroups and Morphisms

by Bret Tilson

1. Definition and Basic Properties	313
2. The Standard Complexity	320
3. Complexity of Morphisms	326
4. Morphism Classes Defined by \mathbf{S} -Varieties	331
5. The Main Theorems of Complexity	337
6. Examples	339
7. Complexity of Projections	351
8. The Derived Semigroup of a Morphism	356
9. The Rhodes Expansion	361

Contents

10. Proof of the Ideal Theorem	362
11. Construction of the Rhodes Expansion	367
12. \hat{S} Is Fine	372
13. Proof of Property (9.6)	375
14. Problems, Conjectures, and Further Results	379
References	382
INDEX	385