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# Random Media and Boundaries

Unified Theory, Two-Scale Method,  
and Applications

With 22 Figures

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