THE ART OF COMPUTER SYSTEMS PERFORMANCE ANALYSIS
Techniques for Experimental Design, Measurement, Simulation, and Modeling

RAJ JAIN
Digital Equipment Corporation
Littleton, Massachusetts

JOHN WILEY & SONS, INC.
New York / Chichester / Brisbane / Toronto / Singapore
CONTENTS

List of Boxes xcv
List of Case Studies xxvii

PART I AN OVERVIEW OF PERFORMANCE EVALUATION 1

1 Introduction 3
1.1 Outline of Topics, 3
1.2 The Art of Performance Evaluation, 7
1.3 Professional Organizations, Journals, and Conferences, 8
1.4 Performance Projects, 11
Exercise, 12

2 Common Mistakes and How to Avoid Them 14
2.1 Common Mistakes in Performance Evaluation, 14
2.2 A Systematic Approach to Performance Evaluation, 22
Exercises, 28
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.5</td>
<td>Gantt Charts</td>
<td>150</td>
</tr>
<tr>
<td>10.6</td>
<td>Kiviat Graphs</td>
<td>153</td>
</tr>
<tr>
<td>10.7</td>
<td>Schumacher Charts</td>
<td>160</td>
</tr>
<tr>
<td>10.8</td>
<td>Decision Maker's Games</td>
<td>161</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>163</td>
</tr>
<tr>
<td>11</td>
<td>Ratio Games</td>
<td></td>
</tr>
<tr>
<td>11.1</td>
<td>Choosing an Appropriate Base System</td>
<td>165</td>
</tr>
<tr>
<td>11.2</td>
<td>Using an Appropriate Ratio Metric</td>
<td>167</td>
</tr>
<tr>
<td>11.3</td>
<td>Using Relative Performance Enhancement</td>
<td>168</td>
</tr>
<tr>
<td>11.4</td>
<td>Ratio Games with Percentages</td>
<td>169</td>
</tr>
<tr>
<td>11.5</td>
<td>Strategies for Winning a Ratio Game</td>
<td>170</td>
</tr>
<tr>
<td>11.6</td>
<td>Correct Analysis</td>
<td>174</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>174</td>
</tr>
<tr>
<td></td>
<td>Further Reading for Part II</td>
<td>175</td>
</tr>
<tr>
<td>12</td>
<td>Summarizing Measured Data</td>
<td></td>
</tr>
<tr>
<td>12.1</td>
<td>Basic Probability and Statistics Concepts</td>
<td>179</td>
</tr>
<tr>
<td>12.2</td>
<td>Summarizing Data by a Single Number</td>
<td>182</td>
</tr>
<tr>
<td>12.3</td>
<td>Selecting among the Mean, Median, and Mode</td>
<td>183</td>
</tr>
<tr>
<td>12.4</td>
<td>Common Misuses of Means</td>
<td>186</td>
</tr>
<tr>
<td>12.5</td>
<td>Geometric Mean</td>
<td>187</td>
</tr>
<tr>
<td>12.6</td>
<td>Harmonic Mean</td>
<td>188</td>
</tr>
<tr>
<td>12.7</td>
<td>Mean of a Ratio</td>
<td>189</td>
</tr>
<tr>
<td>12.8</td>
<td>Summarizing Variability</td>
<td>192</td>
</tr>
<tr>
<td>12.9</td>
<td>Selecting the Index of Dispersion</td>
<td>195</td>
</tr>
<tr>
<td>12.10</td>
<td>Determining Distribution of Data</td>
<td>196</td>
</tr>
<tr>
<td></td>
<td>Exercises</td>
<td>200</td>
</tr>
<tr>
<td>13</td>
<td>Comparing Systems Using Sample Data</td>
<td></td>
</tr>
<tr>
<td>13.1</td>
<td>Sample versus Population</td>
<td>203</td>
</tr>
<tr>
<td>13.2</td>
<td>Confidence Interval for the Mean</td>
<td>204</td>
</tr>
<tr>
<td>13.3</td>
<td>Testing for a Zero Mean</td>
<td>207</td>
</tr>
<tr>
<td>13.4</td>
<td>Comparing Two Alternatives</td>
<td>208</td>
</tr>
<tr>
<td>13.5</td>
<td>What Confidence Level to Use</td>
<td>212</td>
</tr>
</tbody>
</table>
13.6 Hypothesis Testing versus Confidence Intervals, 213
13.7 One-Sided Confidence Intervals, 214
13.8 Confidence Intervals for Proportions, 215
13.9 Determining Sample Size, 216
Exercises, 218

14 Simple Linear Regression Models
14.1 Definition of a Good Model, 222
14.2 Estimation of Model Parameters, 223
14.3 Allocation of Variation, 226
14.4 Standard Deviation of Errors, 228
14.5 Confidence Intervals for Regression Parameters, 229
14.6 Confidence Intervals for Predictions, 232
14.7 Visual Tests for Verifying the Regression Assumptions, 234
Exercises, 241

15 Other Regression Models
15.1 Multiple Linear Regression Models, 245
15.2 Regression with Categorical Predictors, 254
15.3 Curvilinear Regression, 257
15.4 Transformations, 259
15.5 Outliers, 265
15.6 Common Mistakes in Regression, 266
Exercises, 270

Further Reading for Part III

PART IV EXPERIMENTAL DESIGN AND ANALYSIS

16 Introduction to Experimental Design
16.1 Terminology, 275
16.2 Common Mistakes in Experimentation, 278
16.3 Types of Experimental Designs, 279
Exercise, 282
17 $2^k$ Factorial Designs 283
  17.1 $2^2$ Factorial Designs, 284
  17.2 Computation of Effects, 285
  17.3 Sign Table Method for Calculating Effects, 286
  17.4 Allocation of Variation, 286
  17.5 General $2^k$ Factorial Designs, 291
      Exercise, 292

18 $2^kr$ Factorial Designs with Replications 293
  18.1 $2^r$ Factorial Designs, 293
  18.2 Computation of Effects, 294
  18.3 Estimation of Experimental Errors, 294
  18.4 Allocation of Variation, 295
  18.5 Confidence Intervals for Effects, 298
  18.6 Confidence Intervals for Predicted Responses, 299
  18.7 Visual Tests for Verifying the Assumptions, 302
  18.8 Multiplicative Models for $2^r$ Experiments, 303
  18.9 General $2^kr$ Factorial Design, 308
      Exercise, 313

19 $2^{k-p}$ Fractional Factorial Designs 314
  19.1 Preparing the Sign Table for a $2^{k-p}$ Design, 316
  19.2 Confounding, 318
  19.3 Algebra of Confounding, 320
  19.4 Design Resolution, 321
      Exercises, 326

20 One-Factor Experiments 327
  20.1 Model, 327
  20.2 Computation of Effects, 328
  20.3 Estimating Experimental Errors, 330
  20.4 Allocation of Variation, 331
  20.5 Analysis of Variance, 332
  20.6 Visual Diagnostic Tests, 334
  20.7 Confidence Intervals for Effects, 335
  20.8 Unequal Sample Sizes, 337
      Exercise, 342
CONTENTS

21 Two-Factor Full Factorial Design without Replications 343
  21.1 Model, 344
  21.2 Computation of Effects, 344
  21.3 Estimating Experimental Errors, 346
  21.4 Allocation of Variation, 347
  21.5 Analysis of Variance, 348
  21.6 Confidence Intervals for Effects, 351
  21.7 Multiplicative Models for Two-Factor Experiments, 353
  21.8 Missing Observations, 360
      Exercises, 367

22 Two-Factor Full Factorial Design with Replications 368
  22.1 Model, 368
  22.2 Computation of Effects, 369
  22.3 Computation of Errors, 372
  22.4 Allocation of Variation, 372
  22.5 Analysis of Variance, 374
  22.6 Confidence Intervals for Effects, 374
      Exercise, 379

23 General Full Factorial Designs with $k$ Factors 381
  23.1 Model, 381
  23.2 Analysis of a General Design, 382
  23.3 Informal Methods, 386
      Exercises, 389

Further Reading for Part IV 390

PART V SIMULATION 391

24 Introduction to Simulation 393
  24.1 Common Mistakes in Simulation, 394
  24.2 Other Causes of Simulation Analysis Failure, 395
  24.3 Terminology, 398
  24.4 Selecting a Language for Simulation, 401
  24.5 Types of Simulations, 403
  24.6 Event-Set Algorithms, 408
      Exercises, 411
25 Analysis of Simulation Results

25.1 Model Verification Techniques, 413
25.2 Model Validation Techniques, 420
25.3 Transient Removal, 423
25.4 Terminating Simulations, 428
25.5 Stopping Criteria: Variance Estimation, 430
25.6 Variance Reduction, 436
   Exercises, 436

26 Random-Number Generation

26.1 Desired Properties of a Good Generator, 437
26.2 Linear-Congruential Generators, 439
26.3 Tausworthe Generators, 444
26.4 Extended Fibonacci Generators, 450
26.5 Combined Generators, 450
26.6 A Survey of Random-Number Generators, 452
26.7 Seed Selection, 453
26.8 Myths about Random-Number Generation, 455
   Exercises, 458

27 Testing Random-Number Generators

27.1 Chi-Square Test, 461
27.2 Kolmogorov-Smirnov Test, 462
27.3 Serial-Correlation Test, 465
27.4 Two-Level Tests, 466
27.5 $k$-Dimensional Uniformity or $k$-Distributivity, 467
27.6 Serial Test, 468
27.7 Spectral Test, 470
   Exercises, 473

28 Random-Variate Generation

28.1 Inverse Transformation, 474
28.2 Rejection, 476
28.3 Composition, 478
28.4 Convolution, 479
28.5 Characterization, 480
   Exercise, 482
29 Commonly Used Distributions 483

29.1 Bernoulli Distribution, 483
29.2 Beta Distribution, 484
29.3 Binomial Distribution, 485
29.4 Chi-Square Distribution, 486
29.5 Erlang Distribution, 487
29.6 Exponential Distribution, 488
29.7 $F$ Distribution, 489
29.8 Gamma Distribution, 490
29.9 Geometric Distribution, 491
29.10 Lognormal Distribution, 492
29.11 Negative Binomial Distribution, 492
29.12 Normal Distribution, 493
29.13 Pareto Distribution, 495
29.14 Pascal Distribution, 495
29.15 Poisson Distribution, 496
29.16 Student's $t$ Distribution, 497
29.17 Uniform Distribution (Continuous), 497
29.18 Uniform Distribution (Discrete), 498
29.19 Weibull Distribution, 499
29.20 Relationships among Distributions, 499

Exercises, 501

Further Reading for Part V 502

Current Areas of Research in Simulation, 503

PART VI QUEUEING MODELS 505

30 Introduction to Queueing Theory 507

30.1 Queueing Notation, 507
30.2 Rules for All Queues, 510
30.3 Little's Law, 513
30.4 Types of Stochastic Processes, 515

Exercises, 518
31 Analysis of a Single Queue

31.1 Birth-Death Processes, 519
31.2 M/M/1 Queue, 522
31.3 M/M/m Queue, 527
31.4 M/M/m/B Queue with Finite Buffers, 534
31.5 Results for Other Queueing Systems, 540
Exercises, 545

32 Queueing Networks

32.1 Open and Closed Queueing Networks, 547
32.2 Product Form Networks, 548
32.3 Queueing Network Models of Computer Systems, 552
Exercise, 554

33 Operational Laws

33.1 Utilization Law, 556
33.2 Forced Flow Law, 557
33.3 Little's Law, 560
33.4 General Response Time Law, 561
33.5 Interactive Response Time Law, 563
33.6 Bottleneck Analysis, 563
Exercises, 568

34 Mean-Value Analysis and Related Techniques

34.1 Analysis of Open Queueing Networks, 570
34.2 Mean-Value Analysis, 575
34.3 Approximate MVA, 579
34.4 Balanced Job Bounds, 585
Exercises, 591

35 Convolution Algorithm

35.1 Distribution of Jobs in a System, 593
35.2 Convolution Algorithm for Computing $G(N)$, 595
35.3 Computing Performance Using $G(N)$, 598
35.4 Timesharing Systems, 602
Exercises, 607
36 Hierarchical Decomposition of Large Queueing Networks 608
36.1 Load-dependent Service Centers, 608
36.2 Hierarchical Decomposition, 613
36.3 Limitations of Queueing Theory, 620
Exercises, 622

Further Reading for Part VI 624
Symbols Frequently Used in Queueing Analysis, 624

Appendix A Statistical Tables 627
A.1 Area of the Unit Normal Distribution, 628
A.2 Quantiles of the Unit Normal Distribution, 629
A.3 Commonly Used Normal Quantiles, 630
A.4 Quantiles of the $t$ Distribution, 631
A.5 Quantiles of the Chi-Square Distribution, 632
A.6 90-Percentiles of the $F(n,m)$ Distribution, 634
A.7 95-Percentiles of the $F(n,m)$ Distribution, 635
A.8 99-Percentiles of the $F(n,m)$ Distribution, 636
A.9 Quantiles of the K–S Distribution, 637
A.10 Approximation Formulas for Statistical Tables, 638

Solutions to Selected Exercises 639

References 651

Author Index 661

Subject Index 665