

Murray A. Marshall

# Spaces of Orderings and Abstract Real Spectra



Springer

# Table of Contents

<b>Introduction</b> .....	1
<b>1. Orderings on Fields</b>	
1.1 Lang's homomorphism theorem .....	5
1.2 Characters on groups of exponent 2 .....	8
1.3 Orderings and real places .....	11
1.4 Examples .....	14
<b>2. Spaces of Orderings</b>	
2.1 Axioms for spaces of orderings .....	19
2.2 Quadratic forms and the Witt ring .....	24
2.3 Pfister's local-global principle .....	30
2.4 Subspaces and preorderings .....	32
<b>3. Fans and the Representation Theorem</b>	
3.1 Fans .....	37
3.2 The representation theorem .....	40
3.3 SAP spaces .....	44
3.4 The stability index .....	47
3.5 Fans and real places .....	50
3.6 Trivialization theorem for fans .....	54
<b>4. P-Structures, Connected Components and the Isotropy Theorem</b>	
4.1 Two constructions .....	61
4.2 Spaces of orderings of finite chain length .....	65
4.3 The isotropy theorem .....	67
4.4 Proofs in the field case .....	70
4.5 Lemmas concerning the subspaces $X_t$ , $t \in \chi(G)$ .....	73
4.6 Connected components and P-structures .....	76
4.7 The chain length theorem .....	80

<b>5. The Real Spectrum of a Ring</b>	
5.1 Orderings on rings	83
5.2 Examples	86
5.3 Constructible sets and semi-algebraic sets	89
5.4 Nullstellensatz and Positivstellensatz	91
5.5 Value sets of quadratic forms	95
<b>6. Abstract Real Spectra</b>	
6.1 Axioms for abstract real spectra	99
6.2 Properties of value sets	105
6.3 The spectral topology	110
6.4 More on specialization	114
6.5 Zariski topology and ideals	117
6.6 Subspaces and preorderings	121
6.7 Bröcker's weak local-global principle	127
6.8 Hörmander-Lojasiewicz inequality	129
<b>7. Minimal Generation of Constructible Sets</b>	
7.1 Basic sets	133
7.2 Minimal generation of basic open sets	134
7.3 Dimension	138
7.4 Minimal generation of basic closed sets	140
7.5 More on basic sets	142
7.6 Zero dimensional case	144
7.7 Separating families, etc.	146
<b>8. Structure Results and Realization Results</b>	
8.1 Semi-local case	151
8.2 Bröcker's Axioms	155
8.3 Representation theorems	160
8.4 Behaviour of fans under specialization	163
8.5 Classification of abstract real spectra of finite chain length	164
8.6 Real places and P-structures	168
8.7 Realizing abstract real spectra of finite chain length	172
8.8 Real closed abstract real spectra	176
8.9 Classification of zero dimensional abstract real spectra	179
<b>References</b>	183
<b>Subject Index</b>	187
<b>Symbol Index</b>	189