

Theory of Value, Prices and Accumulation

A mathematical integration of Marx, von Neumann and Sraffa

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