

# Lecture Notes in Economics and Mathematical Systems

Managing Editors: M. Beckmann and W. Krelle

245

Jochen E.M. Wilhelm

|                                 |           |
|---------------------------------|-----------|
| TECHNISCHE HOCHSCHULE DARMSTADT |           |
| Fachbereich 1                   |           |
| Gesamtbibliothek                |           |
| Betriebswirtschaftslehre        |           |
| Inventar-Nr. :                  | 38.107    |
| Abstell-Nr. :                   | A 14/1147 |
| Sachgebiete :                   | 1.6.8.1.3 |

## Arbitrage Theory

Introductory Lectures on Arbitrage-Based  
Financial Asset Pricing



BWL TU Darmstadt



54503733

Springer-Verlag  
Berlin Heidelberg New York Tokyo

# TABLE OF CONTENTS

|        |                                                                                            |    |
|--------|--------------------------------------------------------------------------------------------|----|
| 0.     | <u>INTRODUCTION</u> .....                                                                  | 1  |
| 1.     | <u>THE LINEAR STRUCTURE OF CAPITAL ASSET PRICING MODELS</u> .....                          | 3  |
| 1.1.   | THE BASIC IDEA OF THEORIES OF FINANCIAL ASSET PRICES.                                      | 3  |
| 1.2.   | CASH-FLOW ANALYSIS .....                                                                   | 5  |
| 1.3.   | THE CLASSICAL CAPM .....                                                                   | 6  |
| 1.3.1. | Some Assumptions and Notations .....                                                       | 6  |
| 1.3.2. | Mean-Variance Efficiency .....                                                             | 9  |
| 1.3.3. | The Valuation Formula and Related Issues .....                                             | 14 |
| 1.3.4. | The CAPM Structure of Asset Returns .....                                                  | 18 |
| 1.3.5. | Synopsis of Results in the CAPM Theory .....                                               | 19 |
| 1.4.   | THE CAPM-VERSION BY BLACK .....                                                            | 20 |
| 1.4.1. | The Derivation of Valuation Formulas .....                                                 | 20 |
| 1.4.2. | The Structure of Asset Returns in BLACK's Model.                                           | 24 |
| 1.5.   | THE CAPM-VERSION WITH NON-MARKETABLE INCOME .....                                          | 25 |
| 1.5.1. | The Derivation of Valuation Formulas .....                                                 | 25 |
| 1.5.2. | The Individual Portfolio Structure .....                                                   | 28 |
| 1.6.   | THE SEGMENTED MARKETS MODEL .....                                                          | 31 |
| 1.7.   | SYNOPSIS OF THE MAIN RESULTS .....                                                         | 35 |
| 1.8.   | THE ROLE ARBITRAGE PLAYED IN THE DESCRIBED ASSET<br>PRICING THEORIES .....                 | 36 |
| 2.     | <u>TAXONOMY OF ARBITRAGE IN FINANCIAL MARKETS</u> .....                                    | 40 |
| 3.     | <u>MODELLING AND FIRST CONSEQUENCES OF ARBITRAGE AND<br/>NO-ARBITRAGE CONDITIONS</u> ..... | 43 |
| 3.1.   | NOTATIONAL CONVENTIONS; ARBITRATION AND SPREADS .....                                      | 43 |
| 3.2.   | ARBITRATION AND NO SPREADS: RESULTS WITHOUT TRANS-<br>ACTION COSTS .....                   | 45 |
| 3.3.   | FREE LUNCHES .....                                                                         | 50 |
| 3.3.1. | Concepts and Definitions .....                                                             | 50 |
| 3.3.2. | Transaction Costs and Free Lunches .....                                                   | 54 |

|        |                                                                                                           |     |
|--------|-----------------------------------------------------------------------------------------------------------|-----|
| 4.     | <u>NO-ARBITRAGE CONDITIONS AND THE STRUCTURE OF PRICE SYSTEMS..</u>                                       | 60  |
| 4.1.   | THE LAW OF ONE PRICE .....                                                                                | 60  |
| 4.2.   | FREE LUNCHES AND THE LAW OF ONE PRICE .....                                                               | 64  |
| 4.3.   | VALUATION BY ARBITRAGE .....                                                                              | 65  |
| 4.3.1. | The General Concept .....                                                                                 | 65  |
| 4.3.2. | An Example: Two-State Option Pricing .....                                                                | 68  |
| 4.4.   | THE STRUCTURE OF ASSET PRICES UNDER NO-ARBITRAGE<br>CONDITIONS .....                                      | 71  |
| 4.4.1. | The Statement of No-Arbitrage Conditions .....                                                            | 71  |
| 4.4.2. | The Implications of "No Free Lunches" for the<br>Two-State Option Pricing .....                           | 75  |
| 4.4.3. | The One-Period Case .....                                                                                 | 76  |
| 4.4.4. | The Multiperiod Case .....                                                                                | 79  |
| 5.     | <u>THE STRUCTURE OF ASSET RETURNS AND MEAN VARIANCE EFFICIENCY<br/>UNDER NO-ARBITRAGE CONDITIONS.....</u> | 81  |
| 5.1.   | THE STRUCTURE OF ASSET RETURNS .....                                                                      | 81  |
| 5.2.   | MEAN-VARIANCE EFFICIENCY .....                                                                            | 85  |
| 6.     | <u>SOME SELECTED APPLICATIONS .....</u>                                                                   | 90  |
| 6.1.   | OPTIONS .....                                                                                             | 90  |
| 6.1.1. | No Early Exercise of an American Call .....                                                               | 90  |
| 6.1.2. | Put-Call-Parity .....                                                                                     | 91  |
| 6.1.3. | The Valuation of Contingent Claims in Discrete<br>Time .....                                              | 92  |
| 6.2.   | FORWARD AND FUTURES CONTRACTS .....                                                                       | 95  |
| 6.2.1. | Interest Rate Parity Theory of Foreign Exchange<br>Rates .....                                            | 95  |
| 6.2.2. | Forward and Futures Prices .....                                                                          | 96  |
| 6.3.   | CORPORATE FINANCIAL POLICY.....                                                                           | 99  |
| 6.3.1. | The Valuation of Levered Firms .....                                                                      | 99  |
| 6.3.2. | The FISHER Separation Under Uncertainty .....                                                             | 100 |
| 6.4.   | ARBITRAGE THEORY AND ROSS's ARBITRAGE PRICING THEORY.                                                     | 103 |

|                                               |     |
|-----------------------------------------------|-----|
| <u>LIST OF ASSUMPTIONS</u> .....              | 108 |
| <u>INDEX OF FREQUENTLY USED SYMBOLS</u> ..... | 109 |
| <u>REFERENCES</u> .....                       | 112 |