

AN OUTLINE OF MATHEMATICAL LOGIC

*Fundamental Results and Notions
Explained with All Details*

by

ANDRZEJ GRZEGORCZYK

Warsaw University

Technische Hochschule Darmstadt
FACHBEREICH INFORMATIK

B I B L I O T H E K

Inventar-Nr.: 2147

Sachgebiete:

Standort:



D. REIDEL PUBLISHING COMPANY
DORDRECHT-HOLLAND/BOSTON-U.S.A.

PWN—POLISH SCIENTIFIC PUBLISHERS
WARSZAWA

CONTENTS

PREFACE	VII
-------------------	-----

INTRODUCTION TO THE PROBLEMS OF THE FOUNDATIONS OF MATHEMATICS	1
1. Mathematical Domains	1
2. Examples of Mathematical Domains	5
3. Selected Kinds of Relations and Functions	8
4. Logical Analysis of Mathematical Concepts	18
5. Zermelo's Set Theory	23
6. Set-Theoretical Approach to Relations and Functions	34
7. The Genetic Construction of Natural Numbers	37
8. Expansion of the Concept of Number	44
9. Construction of New Mathematical Domains	49
10. Subdomains, Homomorphisms, Isomorphisms	56
11. Products. Real Numbers	62

CHAPTER I. THE CLASSICAL LOGICAL CALCULUS

1. The Classical Characteristics of the Sentential Connectives	65
2. Tautologies in the Classical Sentential Calculus and Their Applications to Certain Mathematical Considerations	77
3. An Axiomatic Approach to the Sentential Calculus	96
4. The Classical Concept of Quantifier	116
5. The Predicate Calculus in the Traditional Interpretation	131
6. Reduction of Quantifier Rules to Axioms. c.l.c Tautologies True in the Empty Domain	154
7. The Concepts of Consequence and Theory. Applications of the Logical Calculus to the Formalization of Mathematical Theories	185
8. The Logical Functional Calculus L^* and Its Applications to the Formalization of Theories with Functions	203

CONTENTS

9. Certain Syntactic Properties of the Classical Logical Calculus	214
10. On Definitions	236

CHAPTER II. MODELS OF AXIOMATIC THEORIES

1. The Concept of Satisfaction	264
2. The Concepts of Truth and Model. The Properties of the Set of Sentences True in a Model	286
3. Existence of ω -complete Extensions and Denumerable Models	311
4. Some Other Concepts and Results in Model Theory	324
5. Skolem's Elimination of Quantifiers, Consistency of Compound Theories and Interpolation Theorems	351
6. Definability	388

CHAPTER III. LOGICAL HIERARCHY OF CONCEPTS

1. The Concept of Effectiveness in Arithmetic	396
2. Some Properties of Computable Functions	417
3. Effectiveness of Methods of Proof	452
4. Representability of Computable Relations in Arithmetic . .	466
5. Problems of Decidability	504
6. Logical Hierarchy of Arithmetic Concepts	519

SUPPLEMENT. A HISTORICAL OUTLINE	565
--	-----

BIBLIOGRAPHY	581
------------------------	-----

INDEX OF SYMBOLS	586
----------------------------	-----

INDEX OF NAMES	588
--------------------------	-----

SUBJECT INDEX	590
-------------------------	-----