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Three Papers on Algebras and Their Representations

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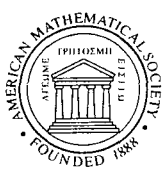

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