

Fourier Analysis

Eric Stade



A JOHN WILEY & SONS, INC., PUBLICATION

Contents

<i>Preface</i>	xv
<i>Introduction</i>	xix
1 <i>Fourier Coefficients and Fourier Series</i>	1
1.1 <i>Periodic Functions: Beginning Bits</i>	1
1.2 <i>Fourier Coefficients of 2π-Periodic Functions</i>	8
1.3 <i>More on $P = 2\pi$</i>	14
1.4 <i>Pointwise Convergence of Fourier Series: A Theorem</i>	25
1.5 <i>An Application: Evaluation of Infinite Series</i>	31
1.6 <i>Gibbs' Phenomenon</i>	36
1.7 <i>Uniform Convergence of Fourier Series: A Theorem</i>	41
1.8 <i>Derivatives, Antiderivatives, and Fourier Series</i>	47
1.9 <i>Functions of Other Periods $P > 0$</i>	55
1.10 <i>Amplitude, Phase, and Spectra</i>	59
1.11 <i>Functions on Bounded Intervals: Standard Fourier Series</i>	65
1.12 <i>Other Fourier Series for Functions on Bounded Intervals</i>	70
2 <i>Fourier Series and Boundary Value Problems</i>	79

2.1	<i>Steady State Temperatures and “the Fourier Method”</i>	79
2.2	<i>Linear Operators, Homogeneous Equations, and Superposition</i>	88
2.3	<i>Heat Flow in a Bar I: Neumann and Mixed Boundary Conditions</i>	94
2.4	<i>Heat Flow in a Bar II: Other Boundary Conditions</i>	100
2.5	<i>Cylindrical and Polar Coordinates</i>	105
2.6	<i>Spherical Coordinates</i>	111
2.7	<i>The Wave Equation I</i>	115
2.8	<i>The Wave Equation II: Existence and Uniqueness of Solutions</i>	123
2.9	<i>The Wave Equation III: Fourier Versus d’Alembert</i>	127
2.10	<i>The Wave Equation IV: Temporally Constant Inhomogeneity</i>	132
2.11	<i>The Wave Equation V: Temporally Varying Inhomogeneity</i>	136
2.12	<i>The Wave Equation VI: Drumming Up Some Interest</i>	145
2.13	<i>Triple Fourier Series</i>	150
3	<i>L^2 Spaces: Optimal Contexts for Fourier Series</i>	155
3.1	<i>The Mean Square Norm and the Inner Product on $C(\mathbb{T})$</i>	155
3.2	<i>The Vector Space $L^2(\mathbb{T})$</i>	163
3.3	<i>More on $L^2(\mathbb{T})$; the Vector Space $L^1(\mathbb{T})$</i>	175
3.4	<i>Norm Convergence of Fourier Series: A Theorem</i>	179
3.5	<i>More on Integration</i>	182
3.6	<i>Orthogonality, Orthonormality, and Fourier Series</i>	187
3.7	<i>More on the Inner Product</i>	195
3.8	<i>Orthonormal Bases for Product Domains</i>	202
3.9	<i>An Application: The Isoperimetric Problem</i>	207
3.10	<i>What Is $L^2(\mathbb{T})$?</i>	209
4	<i>Sturm-Liouville Problems</i>	217
4.1	<i>Definitions and Basic Properties</i>	217
4.2	<i>Some Boundary Value Problems</i>	227
4.3	<i>Bessel Functions I: Bessel’s Equation of Order n</i>	232
4.4	<i>Bessel Functions II: Fourier-Bessel Series</i>	237
4.5	<i>Bessel Functions III: Boundary Value Problems</i>	242
4.6	<i>Orthogonal Polynomials</i>	247
4.7	<i>More on Legendre Polynomials</i>	252

5	<i>Convolution and the Delta Function: A Splat and a Spike</i>	261
5.1	<i>Convolution: What Is It?</i>	261
5.2	<i>Convolution: When Is It Compactly Supported?</i>	265
5.3	<i>Convolution: When Is It Bounded and Continuous?</i>	269
5.4	<i>Convolution: When Is It Differentiable?</i>	273
5.5	<i>Convolution: An Example</i>	278
5.6	<i>Convolution: When Is It In $L^1(\mathbb{R})$? In $L^2(\mathbb{R})$?</i>	283
5.7	<i>Approximate Identities and the Dirac Delta "Function"</i>	287
6	<i>Fourier Transforms and Fourier Integrals</i>	297
6.1	<i>The Fourier Transform on $L^1(\mathbb{R})$: Basics</i>	297
6.2	<i>More on the Fourier Transform on $L^1(\mathbb{R})$</i>	303
6.3	<i>Low-Impact Fourier Transforms (Integration by Differentiation)</i>	311
6.4	<i>Fourier Inversion on $FL^1(\mathbb{R})$</i>	316
6.5	<i>The Fourier Transform and Fourier Inversion on $L^2(\mathbb{R})$</i>	321
6.6	<i>Fourier Inversion of Piecewise Smooth, Integrable Functions</i>	327
6.7	<i>Fourier Cosine and Sine Transforms</i>	333
6.8	<i>Multivariable Fourier Transforms and Inversion</i>	336
6.9	<i>Tempered Distributions: A Home for the Delta Spike</i>	343
7	<i>Special Topics and Applications</i>	353
7.1	<i>Hermite Functions</i>	353
7.2	<i>Boundary Value Problems</i>	358
7.3	<i>Multidimensional Fourier Transforms and Wave Equations</i>	365
7.4	<i>Bandlimited Functions and the Shannon Sampling Theorem</i>	372
7.5	<i>The Discrete Fourier Transform</i>	379
7.6	<i>The Fast Fourier Transform, or FFT, algorithm</i>	385
7.7	<i>Filtering</i>	390
7.8	<i>Linear Systems; Deconvolution</i>	397
7.9	<i>Fraunhofer Diffraction and Fourier Optics</i>	403
7.10	<i>FT-NMR Spectroscopy</i>	412
8	<i>Local Frequency Analysis and Wavelets</i>	421
8.1	<i>Short-Time, or Windowed, Fourier Transforms</i>	421

8.2	<i>Finite Windows and the Heisenberg Uncertainty Principle</i>	430
8.3	<i>Wavelets and Multiresolution Analyses: Basics</i>	436
8.4	<i>Multiresolution Analyses and Wavelets: A Builder's Guide</i>	443
8.5	<i>Proof of Theorem 8.4.1</i>	457
	<i>Appendix</i>	469
	<i>Références</i>	479
	<i>Index</i>	483