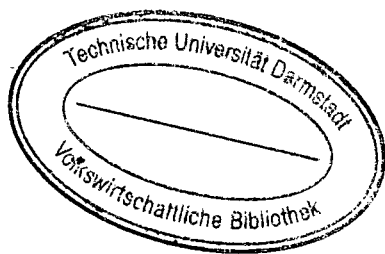


New Approaches to Macroeconomic Modeling

Evolutionary Stochastic Dynamics, Multiple
Equilibria, and Externalities as Field Effects

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CAMBRIDGE
UNIVERSITY PRESS

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