

Introduction to the THEORY OF THIN SHELLS

H. MØLLMANN

*Department of Structural Engineering
Technical University of Denmark*

A Wiley-Interscience Publication

M

Technische Hochschule Darmstadt
Fachbereich Mechanik
Bibliothek
Inv.-Nr. BM 177/82

JOHN WILEY & SONS
Chichester · New York · Brisbane · Toronto · Singapore

CONTENTS

PREFACE	vii
CHAPTER 1 GENERAL THEORY OF SHELLS	1
1.1 Introduction	1
1.2 Some results from the theory of surfaces	2
1.2.1 Lines of curvature as coordinate curves	5
1.2.2 Equations of Codazzi and Gauss	9
1.3 General principles in the theory of shells	11
1.4 Geometrical description	12
1.4.1 The displacements of the middle surface	12
1.4.2 The middle surface strains	16
1.4.3 The rotation vector	18
1.4.4 The bending of the middle surface	21
1.4.5 Equations of compatibility	25
1.5 Statics of the shell	28
1.5.1 Applied loads, contact forces and couples	28
1.5.2 Equations of equilibrium	32
1.5.3 The principle of virtual work	34
1.5.4 Static-geometric analogy, stress functions	42
1.6 The strain energy of the shell, constitutive equations	44
1.7 Edge conditions	48
1.8 Discussion of governing equations	52
Problems	54
CHAPTER 2 SHELLS OF REVOLUTION	57
2.1 Introduction	57
2.2 Governing equations	57
2.2.1 Derivation of governing differential equations	60
2.2.2 Calculation of contact forces and couples and of displacements	65
2.2.3 Particular integral, membrane theory	66

2.2.4 Edge conditions	69
2.2.5 The special case $R_1 = \text{constant}$	72
2.3 Spherical shell	73
2.4 Conical shell	76
2.5 Circular cylindrical shell	82
2.6 Geckeler's method	88
2.7 Numerical example, conical shell	93
2.8 Concluding remarks	100
Problems	100
CHAPTER 3 SHALLOW SHELLS	103
3.1 Simplifications in governing equations	103
3.2 Derivation of governing differential equations	108
3.3 Shallow shell over a rectangular plan	111
3.3.1 The homogeneous equation and the auxiliary equation	116
3.3.2 Solution of the homogeneous equation	119
3.3.3 Particular integral	126
3.3.4 Edge conditions	127
3.4 The Donnell theory for circular cylindrical shells	129
3.5 Shallow spherical shell	131
3.6 Numerical example, shallow shell over a rectangular plan	136
3.7 Numerical example, shallow spherical shell	144
Problems	149
CHAPTER 4 THE THEORY OF SHELLS IN TENSOR NOTATION	153
4.1 Introduction	153
4.2 The theory of surfaces in tensor form	153
4.3 Geometrical description	159
4.3.1 The middle surface strains	159
4.3.2 The rotation vector	160
4.3.3 The bending of the middle surface	161
4.3.4 Equations of compatibility	162
4.4 Statics of the shell	164
4.4.1 Applied loads, contact forces and couples	164
4.4.2 Equations of equilibrium	165
4.4.3 The principle of virtual work	166
4.4.4 Static-geometric analogy, stress functions	168
4.5 Constitutive equations	169
4.6 Edge conditions	170
4.7 Physical components	171
BIBLIOGRAPHY	175
INDEX	179