

# **Algorithmic decision support for train scheduling in a large and highly utilised railway network**

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