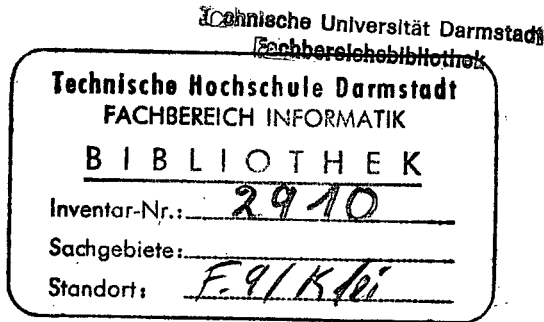


QUEUEING SYSTEMS

VOLUME I: THEORY

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Contents

VOLUME I

PART I: PRELIMINARIES

Chapter 1 Queueing Systems	3
1.1. Systems of Flow	3
1.2. The Specification and Measure of Queueing Systems	8
Chapter 2 Some Important Random Processes	10
2.1. Notation and Structure for Basic Queueing Systems	10
2.2. Definition and Classification of Stochastic Processes	19
2.3. Discrete-Time Markov Chains	26
2.4. Continuous-Time Markov Chains	44
2.5. Birth-Death Processes	53

PART II: ELEMENTARY QUEUEING THEORY

Chapter 3 Birth-Death Queueing Systems in Equilibrium	89
3.1. General Equilibrium Solution	90
3.2. M/M/1: The Classical Queueing System	94
3.3. Discouraged Arrivals	99
3.4. M/M/∞: Responsive Servers (Infinite Number of Servers)	101
3.5. M/M/m: The <i>m</i> -Server Case	102
3.6. M/M/1/K: Finite Storage	103
3.7. M/M/m/m: <i>m</i> -Server Loss Systems.	105
3.8. M/M/1//M: Finite Customer Population—Single Server	106
3.9. M/M/∞//M: Finite Customer Population—"Infinite" Number of Servers	107
3.10. M/M/m/K/M: Finite Population, <i>m</i> -Server Case, Finite Storage	108

Chapter 4 Markovian Queues in Equilibrium	115
4.1. The Equilibrium Equations	115
4.2. The Method of Stages—Erlangian Distribution E.	119
4.3. The Queue $M/E_r/1$	126
4.4. The Queue $E_r/M/1$	130
4.5. Bulk Arrival Systems	134
4.6. Bulk Service Systems	137
4.7. Series-Parallel Stages: Generalizations	139
4.8. Networks of Markovian Queues	147

PART III: INTERMEDIATE QUEUEING THEORY

Chapter 5 The Queue $M/G/1$	167
5.1. The $M/G/1$ System	168
5.2. The Paradox of Residual Life: A Bit of Renewal Theory	169
5.3. The Imbedded Markov Chain	174
5.4. The Transition Probabilities	177
5.5. The Mean Queue Length	180
5.6. Distribution of Number in System	191
5.7. Distribution of Waiting Time	196
5.8. The Busy Period and Its Duration	206
5.9. The Number Served in a Busy Period	216
5.10. From Busy Periods to Waiting Times	219
5.11. Combinatorial Methods	223
5.12. The Takács Integrodifferential Equation	226

Chapter 6 The Queue $G/M/m$	241
6.1. Transition Probabilities for the Imbedded Markov Chain ($G/M/m$)	241
6.2. Conditional Distribution of Queue Size	246
6.3. Conditional Distribution of Waiting Time	250
6.4. The Queue $G/M/1$	251
6.5. The Queue $G/M/m$	253
6.6. The Queue $G/M/2$	256

Chapter 7 The Method of Collective Marks	261
7.1. The Marking of Customers	261
7.2. The Catastrophe Process	267

PART IV: ADVANCED MATERIAL

Chapter 8 The Queue G/G/1	275
8.1. Lindley's Integral Equation	275
8.2. Spectral Solution to Lindley's Integral Equation	283
8.3. Kingman's Algebra for Queues	299
8.4. The Idle Time and Duality	304
Epilogue	319
Appendix I: Transform Theory Refresher: z-Transform and Laplace Transform	
I.1. Why Transforms?	321
I.2. The z -Transform	327
I.3. The Laplace Transform	338
I.4. Use of Transforms in the Solution of Difference and Differential Equations	355
Appendix II: Probability Theory Refresher	
II.1. Rules of the Game	363
II.2. Random Variables	368
II.3. Expectation	377
II.4. Transforms, Generating Functions, and Characteristic Functions	381
II.5. Inequalities and Limit Theorems	388
II.6. Stochastic Processes	393
<i>Glossary of Notation</i>	396
<i>Summary of Important Results</i>	400
<i>Index</i>	411