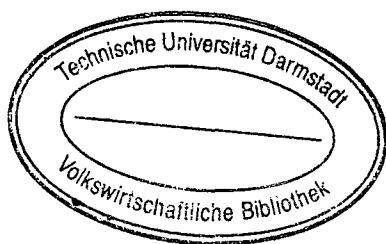


TIME SERIES, UNIT ROOTS, AND COINTEGRATION

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ACADEMIC PRESS

San Diego London Boston New York Sydney Tokyo Toronto

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