

Volker Achim Weberruß

Universality in Statistical Physics and Synergetics

**A Comprehensive Approach
to Modern Theoretical Physics**

with 38 Figures



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