V. Palmov

Vibrations of Elasto-Plastic Bodies

7

Translated by A. Belyaev

With 57 Figures



Contents

7

Translator's Preface

Preface

1	Fou	Foundations and equations of continuum mechanics			
	1.1	Kinematics of a continuous medium	13		
	1.2	Law of conservation of mass	16		
	1.3	Strain and rate of strain	17		
	1.4	Rotation tensor	21		
	1.5	Equations of dynamics for a continuous medium	23		
	1.6	The laws of thermodynamics	24		
	1.7	Thermodynamic processes and constitutive equations	27		
	1.8	Materials with elastic dilatation and the Fourier law of heat			
		conduction	34		
	1.9	Classical materials	38		
		1.9.1 Elastic material	38		
		1.9.2 Viscous materials	41		
		1.9.3 Plastic materials	43		
	1.10	Rheological models of materials. Complex materials	47		
	1.11	Examples of complex materials	58		
		1.11.1 Example 1. The Kelvin-Voigt material	58		
		1.11.2 Example 2. The Maxwell material	60		
		1.11.3 Example 3. The Poynting-Thomson viscoelastic ma-			
		terial	61		

 $\mathbf{5}$

 $\overline{7}$

•

		1.11.4 Example 4. Elastoplastic materials with the Prandtl	
		rheological model	62
		1.11.5 Example 5. A viscoelastoplastic material	64
		1.11.6 Example 6. The Bingham viscoplastic material	66
		1.11.7 Example 7. The Bingham elastoviscoplastic material	67
	1.12	Discussion about the decomposition methods ;	68
	1.13	Linearisation of the equations of continuum mechanics	75
	1.14	Theories of small microplastic deformations	80
		1.14.1 The generalized Prandtl material	80
		1.14.2 The Novozhilov-Kadashevich model	83
2	Plas	sticity theory and internal friction in materials	87
	2.1	Models of elastoplastic materials in the theory of internal	
		friction	87
	2.2	One-dimensional theory of microplasticity	88
	2.3	Harmonic deformation of material	94
	2.4	The choice of the yield stress distribution density $p(h)$. Con-	
		clusions	96
	2.5	Forced vibration of an oscillator with nonlinear internal friction	ı 98
	2.6	Free vibration of an oscillator with nonlinear internal friction	102
3	Thr	ee-dimensional cyclic deformations of elastoplastic ma-	-
	teri	als	105
	3.1	Constitutive equations for elastoplastic materials	105
	3.2	Simple deformation	107
	3.3	Simple cyclic deformation after a simple deformation	113
	3.4	The method of harmonic linearisation for tensor equations .	115
	3.5	Harmonic linearisation of the equation for the elastoplastic	
		material	117
	3.6	The correspondence principle	121
	3.7	Energy dissipation in materials in some particular stress states	s124
		3.7.1 Stress state of pure shear	124
		3.7.2 Uniaxial stress state	124
		3.7.3 Isotropic biaxial stress state	125
		3.7.4 Deformation without shape distortion	126
4	Sing	gle-frequency vibrations in elastoplastic bodies	127
	4.1	The Galerkin method in problems of vibrations of elasto-	
		plastic bodies	127
	4.2	Analysis of experimental data on the energy dissipation of	
		material	132
	4.3	Vibration of a simple torsional system	140
	4.4	Examples of estimation of the complex torsional rigidity	142
		4.4.1 Elliptical cross-section	142
		4.4.2 Rectangular cross-section	145

		4.4.3	Ring cross-section	145					
		4.4.4	Triangular cross-section	145					
	4.5	Torsio	nal vibrations of elastoplastic rods	146					
	4.6	Vibrat	ions of elastoplastic plates	147					
	4.7	Vibrat	ion of rectangular plates with some boundary condition	s150					
		4.7.1	Simply supported plate	150					
		4.7.2	A free square plate	154					
5	Ran	andom deformation of elastoplastic materials 1							
	5.1	The ba	asic results of the theory of stationary random function	s155					
	5.2	The n	nethod of statistical linearisation for analysing defor-						
		matior	ns	157					
	5.3	A gene	eral study of the equation for complex shear modulus	161					
	5.4	Polyha	armonic deformation	166					
	5.5	The si	mple case of broad band deformation	172					
	5.6	On de	termination of the distribution density of defects $p(h)$	175					
6	Rar	odom v	vibrations of elastoplastic bodies	179					
•	6.1	The G	alerkin method in the problem of random vibrations.	179					
	6.2	Rando	om vibrations of an oscillator	185					
	6.3	Analy	sis of experiments of biharmonic vibrations	188					
	6.4	Differe	ential equations of vibration in thin plates	195					
	6.5	Vibrat	tions in an infinite thin plate under broad band random	200					
	0.0	loadin		199					
	6.6	Vibrat	tions in an infinite plate subjected to spatial white						
		noise l	loading	202					
	6.7	Actior	n of a wave load on a plate	208					
	6.8	Differe	ential equations for elastoplastic vibrations in shallow						
		shells	· · · · · · · · · · · · · · · · · · ·	209					
	6.9	Vibrat	tions of a shallow shell under broad band random load	215					
-	Dere			. 999					
1	Pro	pagati	on of vibration in a nonlinear dissipative medium	1223					
	7.1	Fropa	gation of vibration in a linear viscoelastic rod	- 440 - 929					
	1.4	Draien	nent of the problem for a nonlinear dissipative median.	1 202					
	1.5	riopa 701	Dirid plastic materials with linear handening	209					
		7.3.1	Rigid-plastic materials with linear hardening	240					
		7.3.2	Elastoplastic materials with linear hardening	242					
		1.3.3	Elastoplastic materials with an arbitrary hardening	248					
		7.3.4	A viscoelastoplastic material	249					
	7.4	Vibra	tion in a rod with power law elastic and dissipative	- 10					
		charac	teristics	252					
		7.4.1	First example	252					
		7.4.2	Second example	253					
				-00					

12 Contents

	7.5	Propagation of vibration in a nonlinear dissipative rod of finite length	7
	7.6	Propagation of random vibration in a rod with nonlinear properties	6
8	Pro	pagation of vibration in media with complex structure27	7
	8.1	On a model of the medium with complex structure 27	7
	8.2	One-dimensional vibration in the medium with complex struc-	
		ture	0
	8.3	On application of the theory of media with complex structure28	6
	8.4	Theory of vibroconductivity	9
	Ref	erences)5
	\mathbf{Sub}	ject Index)9